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Time correlations in a confined magnetized free-electron gas

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Abstract

The time-dependent pair correlation functions for a degenerate ideal quantum gas of charged particles in a uniform magnetic field are studied on the basis of equilibrium statistics. In particular, the influence of a flat hard wall on the correlations is investigated, both for a perpendicular and a parallel orientation of the wall with respect to the field. The coherent and incoherent parts of the time-dependent structure function in position space are determined from an expansion in terms of the eigenfunctions of the one-particle Hamiltonian. For the bulk of the system, the intermediate scattering function and the dynamical structure factor are derived by taking successive Fourier transforms. In the vicinity of the wall the time-dependent coherent structure function is found to decay faster than in the bulk. For coinciding positions near the wall the form of the structure function turns out to be independent of the orientation of the wall. Numerical results are shown to corroborate these findings.

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1. Introduction

The equilibrium properties of magnetized quantum electron gases are partly determined by physical processes near its boundaries. For instance, the Landau diamagnetic effect of a magnetized free-electron gas is a result of currents flowing near the edge [1]. Hence, it is important to study the influence of a wall on both the static and the dynamic behaviour of these systems. In a recent paper [2] we investigated the static correlations in a fully degenerate magnetized free-electron gas in the vicinity of a wall. In the bulk the decay of the pair correlation function for large separations depends on the orientation of the position difference: for directions orthogonal to the field it is exponentially fast, whereas it is algebraic for directions parallel to the field. If a hard wall parallel to the field is present, the picture changes: the asymptotic behaviour of the pair correlation function for transverse directions changes from exponential to algebraic for any two positions close to the wall.

Further insight into the influence of a wall on the correlation properties is obtained by studying dynamical correlations, which are the subject of the present paper. In the bulk these time correlations yield, after suitable Fourier transforms, the dynamic structure factor, which has been studied quite extensively for various systems, both theoretically and experimentally. For an unmagnetized degenerate free-electron gas its properties were determined several decades ago [3]. The generalization to finite temperatures is available as well [4]. The dynamic structure factor for a magnetized degenerate free-electron gas is somewhat more complicated, as it involves a sum over Landau levels. If a wall is present as well, the translation invariance of the system is lost, so that taking a Fourier transform is inconvenient. Instead, it is more suitable to analyse the dynamical correlations by means of the structure function in configuration space. We will focus on its time dependence, and, in particular, on its decay for large time. As a starting point we shall employ the expansion of the spacetime-dependent structure function in terms of the single-particle eigenfunctions.

The paper is organized as follows. In the next section, we shall define the structure function and the associated time correlation functions. In section 3, we shall derive expressions for the bulk spacetime-dependent structure function of a magnetized degenerate free-electron gas. Some numerical results for this function will be presented as well. The sections 4 and 5 contain an analysis of the intermediate scattering functions and the dynamical structure factor for the bulk, again with some numerical illustrations. In section 6, we shall verify that in the limit of vanishing magnetic field our results reduce to the well-known expressions for the field-free degenerate free-electron gas. Edge effects are the subject of sections 7 and 8. Both the cases of a wall perpendicular to and parallel to the magnetic field will be discussed. The analytical results will be checked numerically.

2. General properties of quantum time correlation functions

The equilibrium time-dependent structure function in position space is defined as

$$S(\mathbf{r},\mathbf{r}',t)=\langle \mathrm{e}^{\mathrm{i}Ht}\psi^\dagger(\mathbf{r})\psi(\mathbf{r})\,\mathrm{e}^{-\mathrm{i}Ht}\psi^\dagger(\mathbf{r}')\psi(\mathbf{r}')\rangle-\langle\psi^\dagger(\mathbf{r})\psi(\mathbf{r})\rangle\langle\psi^\dagger(\mathbf{r}')\psi(\mathbf{r}')\rangle \tag{1}$$

with $\psi(\mathbf{r})$ being the field annihilation operator and H the many-body Hamiltonian. The brackets denote an equilibrium average in the grand-canonical ensemble, with the inverse temperature β and the chemical potential μ. We have chosen units such that $\hbar$ drops out.

For independent particles with Fermi statistics the structure function can be expressed in terms of single-particle wavefunctions as

$$S(\mathbf{r},\mathbf{r}',t)=\sum_j\sum_{j'}\frac{1}{\mathrm{e}^{\beta(E_j-\mu)}+1}\left[1-\frac{1}{\mathrm{e}^{\beta(E_{j'}-\mu)}+1}\right]\varphi_j^*(\mathbf{r})\varphi_j(\mathbf{r}')\varphi_{j'}(\mathbf{r})\varphi_{j'}^*(\mathbf{r}')\,\mathrm{e}^{\mathrm{i}(E_j-E_{j'})t} \tag{2}$$

where $\varphi_j(\mathbf{r})$ are the eigenfunctions of the single-particle Hamiltonian with eigenvalues E_j. The structure function can be written as the sum of an incoherent and a coherent part [5]. These are given as

$$S_{\mathrm{inc}}(\mathbf{r},\mathbf{r}',t)=G^*_{\mu,T}(\mathbf{r},\mathbf{r}',t)G(\mathbf{r},\mathbf{r}',t) \tag{3}$$

$$S_{\mathrm{coh}}(\mathbf{r},\mathbf{r}',t)=-|G_{\mu,T}(\mathbf{r},\mathbf{r}',t)|^2. \tag{4}$$

Here we introduced the time correlation functions $G_{\mu,T}$ and G, which are defined as

$$G_{\mu,T}(\mathbf{r},\mathbf{r}',t)=\sum_j\frac{1}{\mathrm{e}^{\beta(E_j-\mu)}+1}\varphi_j(\mathbf{r})\varphi_j^*(\mathbf{r}')\,\mathrm{e}^{-\mathrm{i}E_jt} \tag{5}$$

$$G(\mathbf{r}, \mathbf{r}', t) = \sum_j \varphi_j(\mathbf{r})\varphi_j^*(\mathbf{r}')\,\mathrm{e}^{-\mathrm{i}E_j t}. \tag{6}$$

The first function depends on μ and T, whereas the second does not. In the static case of vanishing t the correlation function G reduces to $\delta(\mathbf{r} - \mathbf{r}')$.

Our main interest in the following will be in the fully degenerate case with $T = 0$. In that case $G_{\mu,T}$ reduces to

$$G_{\mu,T=0}(\mathbf{r}, \mathbf{r}', t) = \sum_j \theta(\mu - E_j)\varphi_j(\mathbf{r})\varphi_j^*(\mathbf{r}')\,\mathrm{e}^{-\mathrm{i}E_j t} \tag{7}$$

with θ being the step function. The time correlation function for finite T can easily be recovered from that of the fully degenerate case, since one has

$$G_{\mu,T}(\mathbf{r}, \mathbf{r}', t) = \int_0^\infty \mathrm{d}\mu' \frac{1}{\mathrm{e}^{\beta(\mu'-\mu)} + 1} \frac{\mathrm{d}}{\mathrm{d}\mu'} G_{\mu',T=0}(\mathbf{r}, \mathbf{r}', t). \tag{8}$$

The subscript $T = 0$ will often be omitted in the following, so that $G_{\mu,T=0}$ will be written as G_μ.

In uniform systems, or in the bulk of a confined system, the structure function $S(\mathbf{r}, \mathbf{r}', t)$ depends on the position difference $\mathbf{r} - \mathbf{r}'$ only. In this case the intermediate scattering function follows by taking the spatial Fourier transform of S:

$$S(\mathbf{k}, t) = \int \mathrm{d}(\mathbf{r} - \mathbf{r}')\,\mathrm{e}^{-\mathrm{i}\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')} S(\mathbf{r}, \mathbf{r}', t) \tag{9}$$

and similarly for the incoherent and coherent parts. A further Fourier transform leads to the dynamic structure factor:

$$S(\mathbf{k}, \omega) = \int \mathrm{d}t\,\mathrm{e}^{\mathrm{i}\omega t} S(\mathbf{k}, t). \tag{10}$$

Substituting (5) and (6) in (3) and performing the Fourier transforms one gets

$$\begin{aligned} S_{\mathrm{inc}}(\mathbf{k}, \omega) = 2\pi \sum_j \sum_{j'} & \frac{1}{\mathrm{e}^{\beta(E_j-\mu)} + 1} \delta(\omega + E_j - E_{j'}) \\ & \times \int \mathrm{d}(\mathbf{r} - \mathbf{r}')\,\mathrm{e}^{-\mathrm{i}\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')} \varphi_j^*(\mathbf{r})\varphi_j(\mathbf{r}')\varphi_{j'}(\mathbf{r})\varphi_{j'}^*(\mathbf{r}'). \end{aligned} \tag{11}$$

A similar expression can be written for the coherent part. Comparing the two expressions one easily finds the relation

$$S_{\mathrm{coh}}(\mathbf{k}, \omega) = \frac{1}{\mathrm{e}^{\beta\omega} - 1} S_{\mathrm{inc}}(\mathbf{k}, \omega) + \frac{1}{\mathrm{e}^{-\beta\omega} - 1} S_{\mathrm{inc}}(-\mathbf{k}, -\omega). \tag{12}$$

This relation also follows from the principle of detailed balance [6], if the symmetry of the coherent dynamic structure factor under a change of sign of both $\mathbf{k}$ and ω is taken into account.

3. Time correlations in the bulk

We consider a system of charged particles which move in a uniform magnetic field directed along the z-axis. The interaction between the particles is neglected. To describe the magnetic field we adopt the Landau gauge, with vector potential $\boldsymbol{A} = (0, Bx, 0)$. The single-particle Hamiltonian reads

$$H = -\frac{1}{2}\Delta + \mathrm{i}Bx\frac{\partial}{\partial y} + \frac{1}{2}B^2x^2. \tag{13}$$

Units have been chosen such that the charge and the mass of the particles drop out, while c (and $\hbar$) have been put to 1 as well. From now on we will often measure distances in terms of the cyclotron radius $1/\sqrt{B}$. To that end we introduce the reduced variables $\xi = \sqrt{B}x$, $\eta = \sqrt{B}y$ and $\zeta = \sqrt{B}z$.

The normalized eigenfunctions of the Hamiltonian are

$$\psi_{n_y,n_z,n}(\mathbf{r}) = \frac{B^{1/4}}{2^{n/2}\pi^{1/4}(n!)^{1/2}L} H_n(\xi-\kappa_y)\,\mathrm{e}^{\mathrm{i}\kappa_y\eta+\mathrm{i}\kappa_z\zeta-\frac{1}{2}(\xi-\kappa_y)^2}. \tag{14}$$

We impose periodic boundary conditions in the y- and z-directions, with a periodicity length L. The wavevector components are given by $k_i = 2\pi n_i/L$, with integer n_i for $i = y, z$; the reduced wavevector components are $\kappa_i = k_i/\sqrt{B}$. The Hermite polynomials H_n carry the non-negative integer label n. The associated eigenvalue is the sum of the kinetic energy for the z-direction and the Landau-level energy for the transverse directions:

$$E_{n_z,n} = B\left(n+\tfrac{1}{2}\right) + \tfrac{1}{2}k_z^2. \tag{15}$$

The time correlation function G_μ for the fully degenerate system follows by inserting the eigenfunctions and eigenvalues in (7). For large L the summations over n_i may be replaced by integrations over κ_i. As a result we find

$$G_\mu(\mathbf{r},\mathbf{r}',t) = \frac{B^{3/2}}{4\pi^{5/2}}\sum_{n=0}^{\infty}\frac{1}{2^n n!}\,\mathrm{e}^{-\mathrm{i}(n+\frac{1}{2})\tau} J_{\perp,n}(\xi,\xi',\eta,\eta')J_{\parallel,n,\nu}(\zeta,\zeta',\tau). \tag{16}$$

We introduce the reduced time $\tau = Bt$, and the reduced chemical potential $\nu = \mu/B$. The integrals over κ_i are given by

$$J_{\perp,n}(\xi,\xi',\eta,\eta') = \int_{-\infty}^{\infty}\mathrm{d}\kappa_y\, H_n(\xi-\kappa_y)H_n(\xi'-\kappa_y)\,\mathrm{e}^{\mathrm{i}\kappa_y(\eta-\eta')-\frac{1}{2}(\xi-\kappa_y)^2-\frac{1}{2}(\xi'-\kappa_y)^2} \tag{17}$$

$$J_{\parallel,n,\nu}(\zeta,\zeta',\tau) = \int_{-\infty}^{\infty}\mathrm{d}\kappa_z\theta\left(\nu-\frac{1}{2}\kappa_z^2-n-\frac{1}{2}\right)\mathrm{e}^{\mathrm{i}\kappa_z(\zeta-\zeta')-\frac{1}{2}\mathrm{i}\kappa_z^2\tau}. \tag{18}$$

The integration over the transverse wavevector can be carried out by using the identity [7]:

$$\int_{\infty}^{\infty}\mathrm{d}k\, H_n(k+a)H_n(k+b)\,\mathrm{e}^{-k^2} = 2^n\sqrt{\pi}n!L_n(-2ab) \tag{19}$$

for arbitrary complex a and b. One finds

$$J_{\perp,n}(\xi,\xi',\eta,\eta') = 2^n\sqrt{\pi}n!\,\mathrm{e}^{-\frac{1}{4}(\rho_\perp-\rho'_\perp)^2+\frac{1}{2}\mathrm{i}(\xi+\xi')(\eta-\eta')}L_n\left[\tfrac{1}{2}(\boldsymbol{\rho}_\perp-\boldsymbol{\rho}'_\perp)^2\right] \tag{20}$$

with the two-dimensional vectors $\boldsymbol{\rho}_\perp = (\xi,\eta)$ and $\boldsymbol{\rho}'_\perp = (\xi',\eta')$. The integral over the longitudinal wavevector can be rewritten in terms of Fresnel integrals [8] as

$$J_{\parallel,n,\nu}(\zeta,\zeta',\tau) = \theta\left(\nu-n-\frac{1}{2}\right)\sqrt{\frac{\pi}{\tau}}\,\mathrm{e}^{\frac{1}{2}\mathrm{i}(\zeta-\zeta')^2/\tau}F\left[\sqrt{\tau\left(\nu-n-\frac{1}{2}\right)},\frac{\zeta-\zeta'}{\sqrt{2\tau}}\right] \tag{21}$$

with

$$F(a,b) = C(a+b)-\mathrm{i}S(a+b)+C(a-b)-\mathrm{i}S(a-b). \tag{22}$$

Combining (16) with (20) and (21), we get

$$G_\mu(\mathbf{r},\mathbf{r}',t) = \frac{B^{3/2}}{4\pi^{3/2}\tau^{1/2}}\,\mathrm{e}^{-\frac{1}{4}(\rho_\perp-\rho'_\perp)^2+\frac{1}{2}\mathrm{i}(\xi+\xi')(\eta-\eta')+\frac{1}{2}\mathrm{i}(\zeta-\zeta')^2/\tau}$$
$$\times{\sum_n}'\mathrm{e}^{-\mathrm{i}(n+\frac{1}{2})\tau}L_n\left[\frac{1}{2}(\boldsymbol{\rho}_\perp-\boldsymbol{\rho}'_\perp)^2\right]F\left[\sqrt{\tau\left(\nu-n-\frac{1}{2}\right)},\frac{\zeta-\zeta'}{\sqrt{2\tau}}\right]. \tag{23}$$

The prime indicates that the summation is only over those values of n for which $\nu-\left(n+\frac{1}{2}\right)$ is non-negative, i.e. over the Landau levels that are at least partially filled. It should be noted that the right-hand side is continuous as ν equals a half-odd integer, since $F(a, b)$ vanishes for $a = 0$.

The function $G(\mathbf{r}, \mathbf{r}', t)$ can be evaluated along similar lines. Instead of $J_{\|,n,\nu}(\zeta, \zeta', \tau)$ one encounters an integral such as (18), without the step function, which is easily evaluated. As a result we get

$$G(\mathbf{r}, \mathbf{r}', t) = \frac{(1-\mathrm{i})B^{3/2}}{4\pi^{3/2}\tau^{1/2}}\, \mathrm{e}^{-\frac{1}{4}(\rho_\perp-\rho'_\perp)^2+\frac{1}{2}\mathrm{i}(\xi+\xi')(\eta-\eta')+\frac{1}{2}\mathrm{i}(\zeta-\zeta')^2/\tau} \times \sum_{n=0}^{\infty} \mathrm{e}^{-\mathrm{i}(n+\frac{1}{2})\tau} L_n\left[\frac{1}{2}(\rho_\perp - \rho'_\perp)^2\right]. \tag{24}$$

Here the summation extends over all non-negative integers n. We should add a small negative imaginary part to τ to ensure convergence. A simpler form for G is obtained by using the identity [9]

$$\sum_{n=0}^{\infty} L_n(a)b^n = \frac{1}{1-b}\, \mathrm{e}^{-ab/(1-b)} \tag{25}$$

for $|b| < 1$. In this way (24) becomes

$$G(\mathbf{r}, \mathbf{r}', t) = -\frac{(1+\mathrm{i})B^{3/2}}{8\pi^{3/2}\tau^{1/2}\sin\left(\frac{1}{2}\tau\right)}\, \mathrm{e}^{\frac{1}{4}\mathrm{i}\cot(\frac{1}{2}\tau)(\rho_\perp-\rho'_\perp)^2+\frac{1}{2}\mathrm{i}(\xi+\xi')(\eta-\eta')+\frac{1}{2}\mathrm{i}(\zeta-\zeta')^2/\tau}. \tag{26}$$

Clearly, G is singular for τ equal to a multiple of 2π.

Information on the equal-time correlations is obtained by putting $t = 0$. The function G is equal to $\delta(\mathbf{r} - \mathbf{r}')$ for $t = 0$, as is obvious from its definition. To determine the static limit of G_μ one uses the asymptotic expansion [8] for $u \to \infty$:

$$C(u) - \mathrm{i}S(u) \approx \frac{1}{2}(1-\mathrm{i}) + \frac{\mathrm{i}}{\sqrt{2\pi}u}\, \mathrm{e}^{-\mathrm{i}u^2} + \cdots. \tag{27}$$

In this way one finds from (23):

$$G_\mu(\mathbf{r}, \mathbf{r}', t=0) = \frac{B^{3/2}}{2\pi^2|\zeta-\zeta'|}\, \mathrm{e}^{-\frac{1}{4}(\rho_\perp-\rho'_\perp)^2+\frac{1}{2}\mathrm{i}(\xi+\xi')(\eta-\eta')} \times {\sum_n}' L_n\left[\frac{1}{2}(\rho_\perp-\rho'_\perp)^2\right] \sin\left[\sqrt{2\left(\nu - n - \frac{1}{2}\right)}|\zeta-\zeta'|\right]. \tag{28}$$

This result agrees with that found before [2], if spin degeneracy is taken into account.

The behaviour of the time correlation functions depends on the orientation of the position difference vector with respect to the magnetic field. In particular, one may consider purely transverse and purely longitudinal directions separately.

For a purely transverse position difference, e.g. for $\xi \neq \xi'$ with $\eta = \eta'$ and $\zeta = \zeta'$, one finds from (23):

$$G_\mu(\mathbf{r}, \mathbf{r}', t) = \frac{B^{3/2}}{2\pi^{3/2}\tau^{1/2}}\, \mathrm{e}^{-\frac{1}{4}(\xi-\xi')^2} {\sum_n}' \mathrm{e}^{-\mathrm{i}(n+\frac{1}{2})\tau} L_n\left[\frac{1}{2}(\xi-\xi')^2\right] \times \left[C\left(\sqrt{\tau\left(\nu-n-\frac{1}{2}\right)}\right) - \mathrm{i}S\left(\sqrt{\tau\left(\nu-n-\frac{1}{2}\right)}\right)\right]. \tag{29}$$

This expression gets particularly simple if only the lowest Landau level is filled. For $\frac{1}{2} \leqslant \nu \leqslant \frac{3}{2}$ one gets

$$G_\mu(\mathbf{r}, \mathbf{r}', t) = \frac{B^{3/2}}{2\pi^{3/2}\tau^{1/2}}\, \mathrm{e}^{-\frac{1}{4}(\xi-\xi')^2-\frac{1}{2}\mathrm{i}\tau} \left[C\left(\sqrt{\tau\left(\nu-\frac{1}{2}\right)}\right) - \mathrm{i}S\left(\sqrt{\tau\left(\nu-\frac{1}{2}\right)}\right)\right]. \tag{30}$$

The spatial decay is Gaussian, with a Laguerre-polynomial modulation in the general case (29). As a function of time, G_μ is algebraically damped, with a periodic phase factor and a Fresnel-integral modulation. For large t the latter drops out, as follows from (27). Owing to the Laguerre polynomials in (29), the asymptotic form of the coherent structure function $S_{\mathrm{coh}}(\mathbf{r}, \mathbf{r}', t)$ for large t varies with the magnitude of the transverse position difference, even if the Gaussian prefactor is regarded as a trivial normalization factor. As we shall see, this feature disappears when the position difference becomes purely longitudinal.

The behaviour of G for transverse separations is obtained from (26) with $\eta = \eta'$ and $\zeta = \zeta'$ inserted:

$$G(\mathbf{r}, \mathbf{r}', t) = -\frac{(1+\mathrm{i})B^{3/2}}{8\pi^{3/2}\tau^{1/2}\sin\left(\frac{1}{2}\tau\right)}\,\mathrm{e}^{\frac{1}{4}\mathrm{i}\cot(\frac{1}{2}\tau)(\xi-\xi')^2}. \tag{31}$$

The function depends on the position difference through a phase factor only. As to the time dependence, the same singular behaviour for $\tau = 2\pi m$ as in (26) is obtained.

For a purely longitudinal position difference, the behaviour of G_μ is found from (23) by taking $\xi = \xi'$ and $\eta = \eta'$. One gets

$$G_\mu(\mathbf{r}, \mathbf{r}', t) = \frac{B^{3/2}}{4\pi^{3/2}\tau^{1/2}}\,\mathrm{e}^{\frac{1}{2}\mathrm{i}(\zeta-\zeta')^2/\tau}\sum_n{}' \mathrm{e}^{-\mathrm{i}(n+\frac{1}{2})\tau} F\left[\sqrt{\tau\left(\nu - n - \frac{1}{2}\right)}, \frac{\zeta-\zeta'}{\sqrt{2\tau}}\right]. \tag{32}$$

The spatial dependence is no longer Gaussian: it is contained in a phase factor and in the Fresnel integrals. As a function of time, a qualitatively similar behaviour as in the transverse case is found, with an algebraic damping, a periodic phase factor and a Fresnel-integral modulation. For large t the latter disappears, as before, so that then G_μ depends on the position difference through a phase factor only. Hence, for any value of the longitudinal position difference, $S_{\mathrm{coh}}(\mathbf{r}, \mathbf{r}', t)$ has the same asymptotic form for large t.

The behaviour of G for longitudinal separations immediately follows from (26) as

$$G(\mathbf{r}, \mathbf{r}', t) = -\frac{(1+\mathrm{i})B^{3/2}}{8\pi^{3/2}\tau^{1/2}\sin\left(\frac{1}{2}\tau\right)}\,\mathrm{e}^{\frac{1}{2}\mathrm{i}(\zeta-\zeta')^2/\tau}. \tag{33}$$

As in the transverse case, G depends on the position difference through a phase factor only. The time dependence is dominated by the usual singularities.

Finally, let us consider the time dependence of G_μ and G for coinciding positions ($\mathbf{r} = \mathbf{r}'$). It is obtained by putting $\xi = \xi'$ in (29) and (31):

$$G_\mu(\mathbf{r}, \mathbf{r}, t) = \frac{B^{3/2}}{2\pi^{3/2}\tau^{1/2}}\sum_n{}' \mathrm{e}^{-\mathrm{i}(n+\frac{1}{2})\tau}\left[C\left(\sqrt{\tau\left(\nu - n - \frac{1}{2}\right)}\right) - \mathrm{i}S\left(\sqrt{\tau\left(\nu - n - \frac{1}{2}\right)}\right)\right] \tag{34}$$

$$G(\mathbf{r}, \mathbf{r}, t) = -\frac{(1+\mathrm{i})B^{3/2}}{8\pi^{3/2}\tau^{1/2}\sin\left(\frac{1}{2}\tau\right)}. \tag{35}$$

For large t, the function G_μ reduces to

$$G_\mu(\mathbf{r}, \mathbf{r}, t) \approx \frac{(1-\mathrm{i})B^{3/2}}{4\pi^{3/2}\tau^{1/2}}\sum_n{}' \mathrm{e}^{-\mathrm{i}(n+\frac{1}{2})\tau} = \frac{(1-\mathrm{i})B^{3/2}}{4\pi^{3/2}}\frac{\sin\left[\frac{1}{2}(n_0+1)\tau\right]}{\tau^{1/2}\sin\left(\frac{1}{2}\tau\right)}\,\mathrm{e}^{-\frac{1}{2}\mathrm{i}(n_0+1)\tau}. \tag{36}$$

The sine function in the numerator, with n_0 being the label of the highest occupied Landau level, ensures that G_μ is non-singular for $\tau = 2\pi m$, in contrast to G.

In figure 1 we show numerical results for the normalized bulk coherent structure function $S_{\mathrm{coh}}(\mathbf{r}, \mathbf{r}', t)/S_{\mathrm{coh}}(\mathbf{r}, \mathbf{r}', 0)$, at two values of the reduced chemical potential ν. The anisotropy

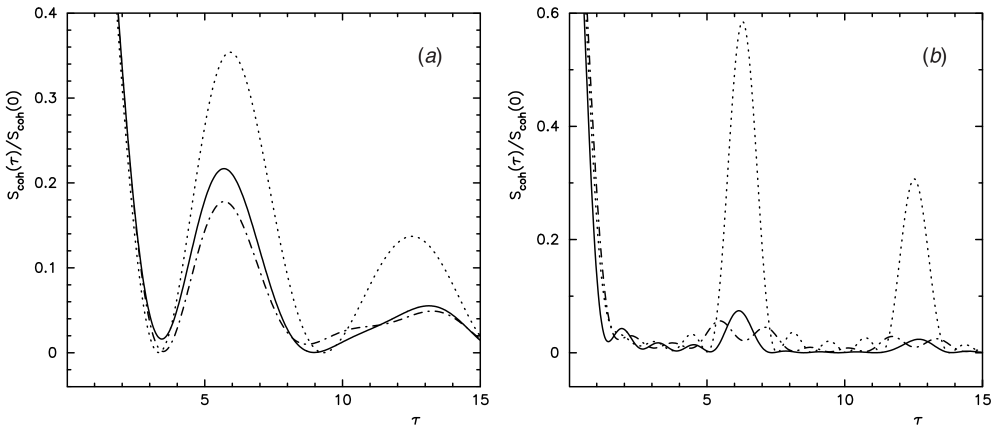

Figure 1. Bulk coherent structure function for $\nu = 2$ (*a*) and $\nu = 5$ (*b*), as a function of time τ. The curves, which are normalized to 1 for $\tau = 0$, represent the structure function for coinciding positions (——), for a longitudinal position difference $|\zeta - \zeta'| = 1$ (······) and for a transverse position difference $|\xi - \xi'| = 1$(— · —).

in the behaviour of the structure function for non-coinciding positions is clearly visible. The curves that correspond to purely longitudinal position differences show characteristic revivals around $\tau = 2\pi m$, with integer m. The revivals are less prominent for purely transverse position differences. These features can easily be understood from the interference effects in the sums in (29) and (32).

4. Bulk intermediate scattering functions

The intermediate scattering function in the bulk follows by Fourier transform with respect to the spatial coordinates. The incoherent part is obtained by inserting (23) and (26) in (9) with (3):

$$S_{\mathrm{inc}}(\mathbf{k}, t) = B^{-3/2} \int \mathrm{d}(\boldsymbol{\rho} - \boldsymbol{\rho}')\, \mathrm{e}^{-\mathrm{i}\boldsymbol{\kappa}\cdot(\boldsymbol{\rho}-\boldsymbol{\rho}')} G^*_\mu(\mathbf{r}, \mathbf{r}', t) G(\mathbf{r}, \mathbf{r}', t)$$
$$= -\frac{(1+\mathrm{i})B^{3/2}}{32\pi^3 \tau \sin\left(\frac{1}{2}\tau\right)} {\sum_n}' \mathrm{e}^{\mathrm{i}(n+\frac{1}{2})\tau} J_{\perp,n}(\boldsymbol{\kappa}_\perp, \tau) J_{\|,n,\nu}(\kappa_z, \tau) \tag{37}$$

with $\boldsymbol{\kappa} = (\boldsymbol{\kappa}_\perp, \kappa_z) = \mathbf{k}/\sqrt{B}$ and $\boldsymbol{\rho} = (\boldsymbol{\rho}_\perp, \zeta) = \sqrt{B}\,\mathbf{r}$. The integrals are defined as

$$J_{\perp,n}(\boldsymbol{\kappa}_\perp, \tau) = \int \mathrm{d}\boldsymbol{\rho}_\perp \exp\left[\frac{1}{2}\rho_\perp^2 \big/ (\mathrm{e}^{-\mathrm{i}\tau} - 1) - \mathrm{i}\boldsymbol{\kappa}_\perp \cdot \boldsymbol{\rho}_\perp\right] L_n\left(\frac{1}{2}\rho_\perp^2\right) \tag{38}$$

$$J_{\|,n,\nu}(\kappa_z, \tau) = \int \mathrm{d}\zeta\, \mathrm{e}^{-\mathrm{i}\kappa_z\zeta} F^*\left[\sqrt{\tau\left(\nu - n - \frac{1}{2}\right)}, \frac{\zeta}{\sqrt{2\tau}}\right]. \tag{39}$$

To evaluate $J_{\perp,n}$, we first perform the angular integral in terms of a Bessel function. Subsequently, we use the identity [7]:

$$\int_0^\infty \mathrm{d}x\, x\, \mathrm{e}^{-\frac{1}{2}ax^2} J_0(xy) L_n\left(\frac{1}{2}x^2\right) = \frac{(a-1)^n}{a^{n+1}}\, \mathrm{e}^{-\frac{1}{2}y^2/a} L_n\left(\frac{y^2}{2a(1-a)}\right) \tag{40}$$

with $\mathrm{Re}\, a > 0$ and $y > 0$. In this way we get

$$J_{\perp,n}(\boldsymbol{\kappa}_\perp, \tau) = 4\pi \mathrm{i}\, \mathrm{e}^{-\mathrm{i}(n+\frac{1}{2})\tau} \sin\left(\tfrac{1}{2}\tau\right) \exp\left[-\tfrac{1}{2}(1-\mathrm{e}^{-\mathrm{i}\tau})\kappa_\perp^2\right] L_n\left[2\sin^2\left(\tfrac{1}{2}\tau\right)\kappa_\perp^2\right]. \tag{41}$$

For the integral over the parallel coordinate we employ the identity

$$\int_{-\infty}^{\infty} \mathrm{d}z\, \mathrm{e}^{-\mathrm{i}kz} F(a, zb) = \frac{2(1-\mathrm{i})}{k}\, \mathrm{e}^{\frac{1}{4}\mathrm{i}k^2/b^2} \sin\left(\frac{ka}{|b|}\right) \tag{42}$$

for real k, a and b, which follows by a partial integration. In this way we find

$$J_{\parallel,n,\nu}(\kappa_z, \tau) = \frac{2(1+\mathrm{i})}{\kappa_z}\, \mathrm{e}^{-\frac{1}{2}\mathrm{i}\kappa_z^2\tau} \sin\left[\sqrt{2\left(\nu - n - \frac{1}{2}\right)}\kappa_z\tau\right]. \tag{43}$$

Combining the above results, we have found the incoherent intermediate scattering function as

$$\begin{aligned} S_{\mathrm{inc}}(\mathbf{k}, t) = {} & \frac{B^{3/2}}{2\pi^2\kappa_z\tau} \exp\left[-\frac{1}{2}\mathrm{i}\kappa_z^2\tau - \frac{1}{2}(1-\mathrm{e}^{-\mathrm{i}\tau})\kappa_\perp^2\right] \\ & \times {\sum_n}' \sin\left[\sqrt{2\left(\nu - n - \frac{1}{2}\right)}\kappa_z\tau\right] L_n\left[2\sin^2\left(\frac{1}{2}\tau\right)\kappa_\perp^2\right]. \end{aligned} \tag{44}$$

For the special cases of purely transverse or purely parallel wavevectors the incoherent intermediate scattering function can be simplified. For $\kappa_z = 0$ it becomes an undamped periodic function of τ with period 2π, whereas for $\kappa_\perp = 0$ it is a decaying function as in the general case.

For $t = 0$ the scattering function reduces to

$$S_{\mathrm{inc}}(\mathbf{k}, t = 0) = \frac{B^{3/2}}{2\pi^2} {\sum_n}' \sqrt{2\left(\nu - n - \frac{1}{2}\right)} \tag{45}$$

which is equal to the bulk particle density n_∞ at the chosen chemical potential. The time derivative at $t = 0$ satisfies the sum rule

$$\left.\frac{\partial}{\partial t} S_{\mathrm{inc}}(\mathbf{k}, t)\right|_{t=0} = -\frac{1}{2}\mathrm{i}\, k^2 n_\infty. \tag{46}$$

We now turn to the coherent intermediate scattering function. By substituting (23) in (9) with (4), we get

$$\begin{aligned} S_{\mathrm{coh}}(\mathbf{k}, t) &= -B^{-3/2} \int \mathrm{d}(\boldsymbol{\rho} - \boldsymbol{\rho}')\, \mathrm{e}^{-\mathrm{i}\boldsymbol{\kappa}\cdot(\boldsymbol{\rho}-\boldsymbol{\rho}')} |G_\mu(\mathbf{r}, \mathbf{r}', t)|^2 \\ &= -\frac{B^{3/2}}{16\pi^3\tau} {\sum_m}' {\sum_n}' \mathrm{e}^{\mathrm{i}(m-n)\tau} J_{\perp,mn}(\boldsymbol{\kappa}_\perp) J_{\parallel,mn,\nu}(\kappa_z, \tau). \end{aligned} \tag{47}$$

The integrals are defined as

$$J_{\perp,mn}(\boldsymbol{\kappa}_\perp) = \int \mathrm{d}\boldsymbol{\rho}_\perp\, \mathrm{e}^{-\frac{1}{2}\rho_\perp^2 - \mathrm{i}\boldsymbol{\kappa}_\perp\cdot\boldsymbol{\rho}_\perp} L_m\left(\frac{1}{2}\rho_\perp^2\right) L_n\left(\frac{1}{2}\rho_\perp^2\right) \tag{48}$$

$$J_{\parallel,mn,\nu}(\kappa_z, \tau) = \int \mathrm{d}\zeta\, \mathrm{e}^{-\mathrm{i}\kappa_z\zeta} F^*\left[\sqrt{\tau\left(\nu - m - \frac{1}{2}\right)}, \frac{\zeta}{\sqrt{2\tau}}\right] F\left[\sqrt{\tau\left(\nu - n - \frac{1}{2}\right)}, \frac{\zeta}{\sqrt{2\tau}}\right]. \tag{49}$$

The angular integral in $J_{\perp,mn}$ leads to a Bessel function as before. The radial integral can be determined by using the identity [7] for $v > 0$:

$$\int_0^\infty \mathrm{d}u\, u\, \mathrm{e}^{-\frac{1}{2}u^2} J_0(uv) L_m\left(\frac{1}{2}u^2\right) L_n\left(\frac{1}{2}u^2\right) = (-1)^{m+n}\,\mathrm{e}^{-\frac{1}{2}v^2} L_m^{-m+n}\left(\frac{1}{2}v^2\right) L_n^{-n+m}\left(\frac{1}{2}v^2\right) \tag{50}$$

which yields

$$J_{\perp,mn}(\boldsymbol{\kappa}_\perp) = 2\pi(-1)^{m+n}\,\mathrm{e}^{-\frac{1}{2}\kappa_\perp^2} L_m^{-m+n}\left(\tfrac{1}{2}\kappa_\perp^2\right) L_n^{-n+m}\left(\tfrac{1}{2}\kappa_\perp^2\right). \tag{51}$$

The two Laguerre polynomials are not independent, as they are related as [10]

$$(-1)^m m!z^{-m} L_m^{-m+n}(z) = (-1)^n n!z^{-n} L_n^{-n+m}(z). \tag{52}$$

Hence, $J_{\perp,mn}(\boldsymbol{\kappa}_\perp)$ is positive for all $m, n \geqslant 0$. For the integral over the parallel coordinate we use the integral identity (A.1) from the appendix. With its help one gets

$$\begin{aligned} J_{\|,mn,\nu}(\kappa_z,\tau) = {} & -\frac{2\mathrm{i}}{\kappa_z}\,\mathrm{e}^{\frac{1}{2}\mathrm{i}\kappa_z^2\tau}\left\{\mathrm{e}^{\mathrm{i}\kappa_z w_n\tau}\theta\left[w_m^2-(\kappa_z+w_n)^2\right]-\mathrm{e}^{-\mathrm{i}\kappa_z w_n\tau}\theta\left[w_m^2-(\kappa_z-w_n)^2\right]\right\} \\ & +\frac{2\mathrm{i}}{\kappa_z}\,\mathrm{e}^{-\frac{1}{2}\mathrm{i}\kappa_z^2\tau}\left\{\mathrm{e}^{-\mathrm{i}\kappa_z w_m\tau}\theta\left[w_n^2-(\kappa_z+w_m)^2\right]-\mathrm{e}^{\mathrm{i}\kappa_z w_m\tau}\theta\left[w_n^2-(\kappa_z-w_m)^2\right]\right\} \end{aligned} \tag{53}$$

with the abbreviation $w_j = \sqrt{2\left(\nu - j - \frac{1}{2}\right)}$.

Substituting (51) and (53) in (47) and employing the symmetry with respect to an interchange of the summations over m and n, we obtain the coherent intermediate scattering function in the form

$$\begin{aligned} S_{\mathrm{coh}}(\mathbf{k},t) = {} & \frac{B^{3/2}}{2\pi^2\kappa_z\tau}\,\mathrm{e}^{-\frac{1}{2}\kappa_\perp^2}\sum_m{}'\sum_n{}'(-1)^{m+n}L_m^{-m+n}\left(\frac{1}{2}\kappa_\perp^2\right)L_n^{-n+m}\left(\frac{1}{2}\kappa_\perp^2\right) \\ & \times\left\{\sin\left[\left(\frac{1}{2}\kappa_z^2-\kappa_z w_m+n-m\right)\tau\right]\theta\left[w_n^2-(\kappa_z-w_m)^2\right]\right. \\ & \left.-\sin\left[\left(\frac{1}{2}\kappa_z^2+\kappa_z w_m+n-m\right)\tau\right]\theta\left[w_n^2-(\kappa_z+w_m)^2\right]\right\}. \end{aligned} \tag{54}$$

For vanishing $\kappa_\perp$ the product of the Laguerre polynomials equals δ_{mn}. As a consequence, the coherent intermediate scattering function gets the simpler form

$$S_{\mathrm{coh}}(\mathbf{k},t) = \frac{B^{3/2}}{2\pi^2|\kappa_z|\tau}\sum_n{}'\sin\left[\left(\frac{1}{2}\kappa_z^2-|\kappa_z|w_m\right)\tau\right]\theta(2w_n-|\kappa_z|). \tag{55}$$

A second special case arises for $\kappa_z = 0$. Expanding the factor between curly brackets in (54) for small κ_z, one finds:

$$\begin{aligned} S_{\mathrm{coh}}(\mathbf{k},t) = {} & -\frac{B^{3/2}}{2\pi^2}\,\mathrm{e}^{-\frac{1}{2}\kappa_\perp^2}\sum_m{}'\sum_n{}'(-1)^{m+n}L_m^{-m+n}\left(\frac{1}{2}\kappa_\perp^2\right)L_n^{-n+m}\left(\frac{1}{2}\kappa_\perp^2\right) \\ & \times\{\cos[(m-n)\tau][(1-\delta_{mn})\theta(m-n)w_m \\ & +(1-\delta_{mn})\theta(n-m)w_n]+\delta_{mn}w_m\}. \end{aligned} \tag{56}$$

In this case the coherent intermediate scattering function is an undamped periodic function of τ.

The static function follows by putting $t = 0$ in (54). Rearranging the terms one may rewrite it as

$$S_{\mathrm{coh}}(\mathbf{k}, t = 0) = -\frac{B^{3/2}}{4\pi^2}\, \mathrm{e}^{-\frac{1}{2}\kappa_\perp^2} {\sum_m}' {\sum_n}' (-1)^{m+n} L_m^{-m+n}\left(\frac{1}{2}\kappa_\perp^2\right) L_n^{-n+m}\left(\frac{1}{2}\kappa_\perp^2\right)$$
$$\times [(w_m + w_n - |\kappa_z|)\theta(w_m + w_n - |\kappa_z|) - (w_m - w_n - |\kappa_z|)$$
$$\times \theta(w_m - w_n - |\kappa_z|) - (w_n - w_m - |\kappa_z|)\theta(w_n - w_m - |\kappa_z|)]. \qquad (57)$$

In contrast to S_{inc}, the coherent intermediate scattering function is an even function of t, as is clear from (54). Hence, the time derivative of S_{coh} at $t = 0$ vanishes, whereas the time derivative of S_{inc} at $t = 0$ is different from 0, as we have seen in (46).

On comparing (44) and (54) one notes that the two parts of the intermediate scattering function are given by rather different expressions. However, they can be made more analogous by employing the identity

$$\sum_{m=0}^{\infty} (-1)^m\, \mathrm{e}^{-\mathrm{i}(m+\frac{1}{2})\tau} L_m^{-m+n}\left(\frac{1}{2}\kappa_\perp^2\right) L_n^{-n+m}\left(\frac{1}{2}\kappa_\perp^2\right)$$
$$= (-1)^n\, \mathrm{e}^{-\mathrm{i}(n+\frac{1}{2})\tau} L_n\left[2\sin^2\left(\frac{1}{2}\tau\right)\kappa_\perp^2\right] \exp\left(\frac{1}{2}\,\mathrm{e}^{-\mathrm{i}\tau}\kappa_\perp^2\right) \qquad (58)$$

which may be proved by using (50) in reverse order, and then (25) and (40). In this way we may write (44) as

$$S_{\mathrm{inc}}(\mathbf{k}, t) = \frac{B^{3/2}}{2\pi^2\kappa_z\tau}\, \mathrm{e}^{-\frac{1}{2}\kappa_\perp^2 - \frac{1}{2}\mathrm{i}\kappa_z^2\tau} \sum_{m=0}^{\infty} {\sum_n}' (-1)^{m+n}\, \mathrm{e}^{\mathrm{i}(n-m)\tau}$$
$$\times L_m^{-m+n}\left(\frac{1}{2}\kappa_\perp^2\right) L_n^{-n+m}\left(\frac{1}{2}\kappa_\perp^2\right) \sin(w_n\kappa_z\tau). \qquad (59)$$

This result could have been obtained in an alternative way by substituting (24) (instead of (26)) in (9) with (3), and taking similar steps as in (47)–(54).

5. Bulk dynamical structure factor

The dynamical structure factor follows from the intermediate scattering function by taking a Fourier transform with respect to t. For the incoherent part of the structure factor we find from (59)

$$S_{\mathrm{inc}}(\mathbf{k}, \omega) = B^{-1} \int_{-\infty}^{\infty} \mathrm{d}\tau\, \mathrm{e}^{\mathrm{i}\varpi\tau} S_{\mathrm{inc}}(\mathbf{k}, t)$$
$$= \frac{B^{1/2}}{2\pi|\kappa_z|}\, \mathrm{e}^{-\frac{1}{2}\kappa_\perp^2} \sum_{m=0}^{\infty} {\sum_n}' (-1)^{m+n} L_m^{-m+n}\left(\frac{1}{2}\kappa_\perp^2\right) L_n^{-n+m}\left(\frac{1}{2}\kappa_\perp^2\right) \theta[\nu - \nu_{mn}(\omega)] \qquad (60)$$

where we introduced the rescaled frequency $\varpi = \omega/B$, and the abbreviation

$$\nu_{mn}(\omega) = n + \tfrac{1}{2} + \tfrac{1}{2}\kappa_z^{-2}\left(\tfrac{1}{2}\kappa_z^2 - \varpi + m - n\right)^2. \qquad (61)$$

The dependence of S_{inc} on ω is determined by the step function. For negative ω it implies that the summation variable m is confined to values below $\nu - \frac{1}{2}$, as is n. For positive ω this need not be the case.

The coherent part of the dynamic structure factor follows from (54) as

$$S_{\rm coh}(\mathbf{k},\omega) = -\frac{B^{1/2}}{2\pi\,|\kappa_z|}\,\mathrm{e}^{-\frac{1}{2}\kappa_\perp^2}\sum_m{}' \sum_n{}' (-1)^{m+n} \times L_m^{-m+n}\left(\frac{1}{2}\kappa_\perp^2\right) L_n^{-n+m}\left(\frac{1}{2}\kappa_\perp^2\right)\theta[\nu - \nu_{mn}(-|\omega|)]. \tag{62}$$

Again the ω-dependence shows up through the step function only.

From (52) it follows that the incoherent part of the structure function is positive for all $\mathbf{k}$ and ω, whereas the coherent part is negative. Furthermore, on comparing (60) and (62) one finds that the two parts of the dynamic structure factor are connected as $S_{\rm coh}(\mathbf{k},\omega) = -S_{\rm inc}(\mathbf{k}, -|\omega|)$. This relation is in agreement with the general identity (12), since the structure factor is symmetric under spatial inversion. It implies that the total dynamic structure factor at $T = 0$ vanishes for negative ω.

If the wavevector is purely transverse, expressions (60) and (62) cannot be used as such. It is more convenient in this case to take the limit $\kappa_z = 0$ in (59) and to use (56) before taking the Fourier transform. In this way one gets

$$S_{\rm inc}(\mathbf{k},\omega) = \frac{B^{1/2}}{\pi}\,\mathrm{e}^{-\frac{1}{2}\kappa_\perp^2}\sum_{m=0}^{\infty}\sum_n{}' (-1)^{m+n} L_m^{-m+n}\left(\frac{1}{2}\kappa_\perp^2\right) L_n^{-n+m}\left(\frac{1}{2}\kappa_\perp^2\right) w_n\,\delta(\varpi - m + n) \tag{63}$$

and $S_{\rm coh}(\mathbf{k},\omega) = -S_{\rm inc}(\mathbf{k}, -|\omega|)$, as before. Both parts of the dynamical structure factor are sums over delta functions, with peaks at the integer resonance frequencies $m - n$.

For wavevectors parallel to the field one finds from (60)

$$S_{\rm inc}(\mathbf{k},\omega) = \frac{B^{1/2}}{2\pi\,|\kappa_z|}\sum_n{}' \theta\,[\nu - \nu_{nn}(\omega)] \tag{64}$$

and an analogous expression for the coherent part. The sum of step functions reaches its maximum value for $\varpi = \frac{1}{2}\kappa_z^2$.

The numerical results in figure 2 show how the coherent dynamic structure factor changes from a function with steps for a purely longitudinal wavevector to a set of discrete spikes for a (nearly) transverse wavevector. A similar behaviour is found for the incoherent dynamic structure factor, as shown in figure 3. For this case the curves are asymmetric with respect to a change of sign of ω.

The sum rules (45) and (46) for the incoherent intermediate scattering function yield

$$\int_{-\infty}^{\infty} \mathrm{d}\omega\, S_{\rm inc}(\mathbf{k},\omega) = 2\pi n_\infty \qquad \int_{-\infty}^{\infty} \mathrm{d}\omega\, \omega S_{\rm inc}(\mathbf{k},\omega) = \pi k^2 n_\infty. \tag{65}$$

The above results for the bulk dynamical structure factor at $T = 0$ can easily be generalized to finite temperatures. In fact, for the incoherent part (11), one may use an identity analogous to (8) to obtain the expression for finite T from that for $T = 0$. Since the chemical potential in (60) only occurs in the step function, the integral over μ' is trivial. In this way one easily arrives at the result for finite temperature:

$$S_{\rm inc}(\mathbf{k},\omega) = \frac{B^{1/2}}{2\pi\,|\kappa_z|}\,\mathrm{e}^{-\frac{1}{2}\kappa_\perp^2}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}(-1)^{m+n} L_m^{-m+n}\left(\frac{1}{2}\kappa_\perp^2\right) L_n^{-n+m}\left(\frac{1}{2}\kappa_\perp^2\right)\frac{1}{\mathrm{e}^{\beta[\nu_{mn}(\omega)-\nu]}+1} \tag{66}$$

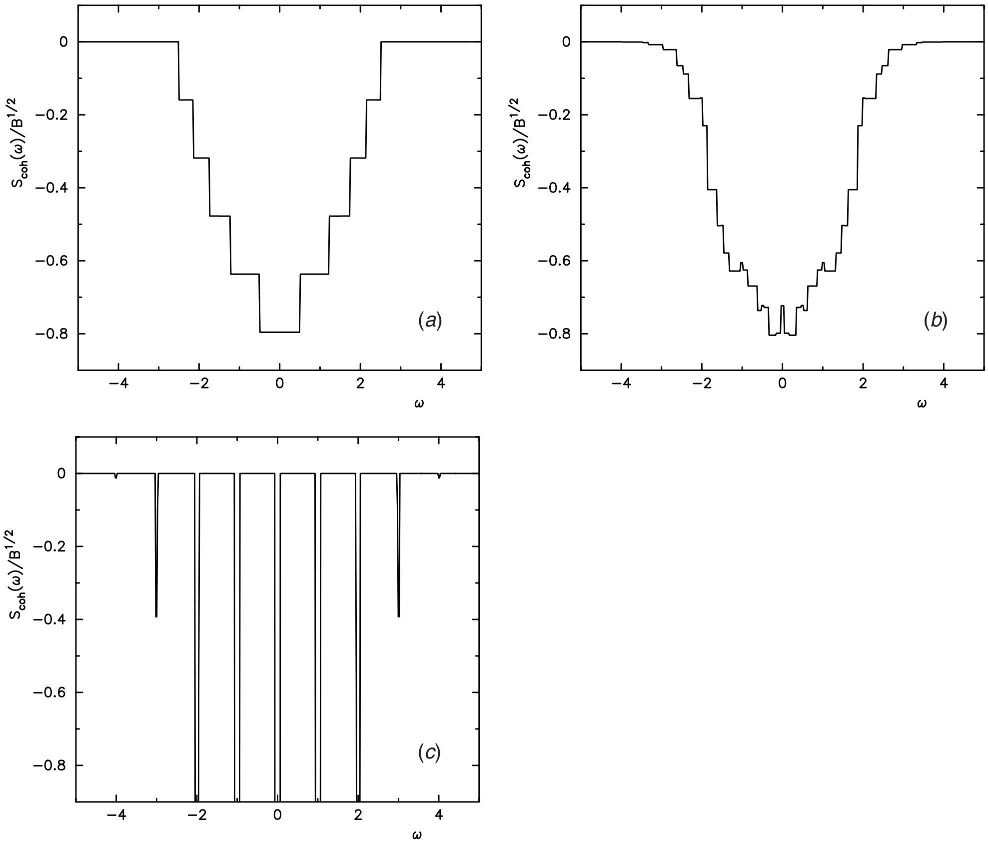

Figure 2. Bulk coherent structure factor $S_{\text{coh}}(\mathbf{k}, \omega)/B^{1/2}$ for $\nu = 5$ and $|\boldsymbol{\kappa}| = 1$, as a function of ϖ. The curves represent the structure factor for $\theta = 0$ (*a*), $\theta = \pi/4$ (*b*) and $\theta = 1.55$ (*c*), with $\theta = \arctan(|\boldsymbol{\kappa}_\perp|/\kappa_z)$.

where the temperature is given in units of B. The coherent part of the dynamic structure factor for finite temperatures follows from (12)

$$S_{\text{coh}}(\mathbf{k}, \omega) = \frac{B^{1/2}}{2\pi|\kappa_z|} \mathrm{e}^{-\frac{1}{2}\kappa_\perp^2} \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} (-1)^{m+n} L_m^{-m+n}\left(\frac{1}{2}\kappa_\perp^2\right) L_n^{-n+m}\left(\frac{1}{2}\kappa_\perp^2\right) \times \left\{ \frac{1}{\mathrm{e}^{\beta\omega}-1} \frac{1}{\mathrm{e}^{\beta[\nu_{mn}(\omega)-\nu]}+1} + \frac{1}{\mathrm{e}^{-\beta\omega}-1} \frac{1}{\mathrm{e}^{\beta[\nu_{mn}(-\omega)-\nu]}+1} \right\}. \tag{67}$$

6. Vanishing magnetic field

In the limit of vanishing magnetic field the results of the previous sections are expected to reduce to the well-known expressions for a non-magnetized degenerate free-electron gas. We may check this by considering the limiting behaviour of the functions F_μ on G in position space.

Expression (23) for G_μ contains a sum that extends to infinity for $B \to 0$. Hence, the order n of the Laguerre polynomial may become large. On the other hand, its argument gets small, since the position variables scale with $\sqrt{B}$. In this regime the Laguerre polynomials

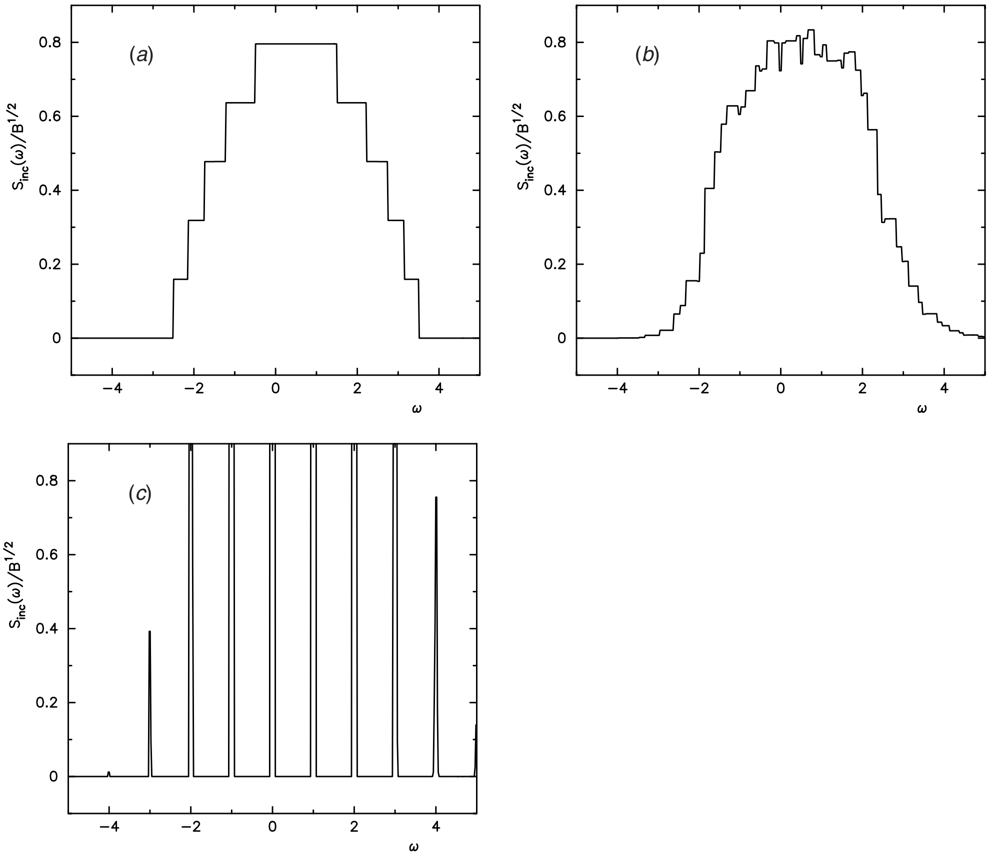

Figure 3. Bulk incoherent structure factor $S_{\mathrm{inc}}(\mathbf{k}, \omega)/B^{1/2}$ for $\nu = 5$ and $|\boldsymbol{\kappa}| = 1$, as a function of ϖ. The curves represent the structure factor for $\theta = 0$ (*a*), $\theta = \pi/4$ (*b*) and $\theta = 1.55$ (*c*), with $\theta = \arctan(|\boldsymbol{\kappa}_\perp|/\kappa_z)$.

may be approximated as [9]

$$L_n(u) \approx \mathrm{e}^{\frac{1}{2}u} J_0(\sqrt{2(2n+1)u}). \tag{68}$$

Putting $nB = v$, we may write the sum in (23) as an integral for small B. Replacing the scaled variables by the original ones, we get for $B = 0$:

$$G_\mu(\mathbf{r}, \mathbf{r}', t) = \frac{1}{4\pi^{3/2} t^{1/2}} \mathrm{e}^{\frac{1}{2}\mathrm{i}(z-z')^2/t} \int_0^\mu \mathrm{d}v\, \mathrm{e}^{-\mathrm{i}vt} J_0(\sqrt{2v}|\mathbf{r}_\perp - \mathbf{r}'_\perp|) F\left(\sqrt{t(\mu - v)}, \frac{z - z'}{\sqrt{2t}}\right). \tag{69}$$

With the use of relation (A.7) of the appendix we obtain

$$\begin{aligned} G_\mu(\mathbf{r}, \mathbf{r}', t) = &-\frac{\mathrm{i}}{4\pi^{3/2} t^{3/2}} \mathrm{e}^{\frac{1}{2}\mathrm{i}|\mathbf{r}-\mathbf{r}'|^2/t} F\left(\sqrt{\mu t}, \frac{|\mathbf{r} - \mathbf{r}'|}{\sqrt{2t}}\right) \\ &+ \frac{\mathrm{i}}{2\pi^2 |\mathbf{r} - \mathbf{r}'| t} \mathrm{e}^{-\mathrm{i}\mu t} \sin(\sqrt{2\mu}|\mathbf{r} - \mathbf{r}'|) \end{aligned} \tag{70}$$

for $B = 0$. For large t the second term dominates. It factorizes in a space-dependent and a time-dependent factor. The latter is proportional to t^{-1} (apart from a t-dependent phase factor). This is in contrast to the behaviour in the case with field. For $\mathbf{r} = \mathbf{r}'$ we have seen in (36) that G_μ has a decay proportional to $t^{-1/2}$ times the ratio of two time-dependent sine functions.

The limiting form of G for vanishing B follows immediately from (26). One finds

$$G(\mathbf{r},\mathbf{r}',t) = -\frac{1+\mathrm{i}}{4\pi^{3/2}t^{3/2}}\,\mathrm{e}^{\frac{1}{2}\mathrm{i}|\mathbf{r}-\mathbf{r}'|^2/t}. \tag{71}$$

It decays algebraically in time, and is free from the singularities that characterize expression (26) for the case with field. For small fields the poles in (26) at $\tau = 2\pi m$, or $t = 2\pi m/B$, with integer $m \neq 0$, shift towards ∞, and disappear for vanishing field. Only the singularity at $t = 0$ remains. It is connected to the delta function singularity of the static function $G(\mathbf{r},\mathbf{r}',t=0)$.

The incoherent and coherent parts of the dynamic structure factor follow by taking Fourier transforms, as we have seen in the previous sections. As a useful check of these results we may verify that the limits of (60) and (62) for vanishing field agree with the well-known results for a field-free degenerate fermion gas. Starting with the incoherent part (60), we first use (50) in reverse order. The resulting integral over u, with $v = \kappa_\perp$, contains a Bessel function $J_0(u\kappa_\perp)$. For a small field, $\kappa_\perp$ becomes large so that effectively only small values of u contribute to the integral. Hence, one may use (68) for the Laguerre polynomials in the integral over u, so that the latter becomes an integral over the product of three Bessel functions. It can be evaluated as a product of two inverse square roots [7]. In this way we have found for large v, and large m, n:

$$(-1)^{m+n}\,\mathrm{e}^{-\frac{1}{2}v^2} L_m^{-m+n}\left(\frac{1}{2}v^2\right) L_n^{-n+m}\left(\frac{1}{2}v^2\right) \approx \frac{2}{\pi\sqrt{16mn-(v^2-2m-2n)^2}}\,\theta[16mn-(v^2-2m-2n)^2]. \tag{72}$$

Employing this asymptotic relation in (60) and replacing the sums over m, n with integrals, by writing $x = mB$ and $y = nB$, we get for $B = 0$

$$\begin{aligned} S_{\mathrm{inc}}(\mathbf{k},\omega) = {}& \frac{1}{2\pi^2|k_z|}\int_0^\infty \mathrm{d}x \int_0^\mu \mathrm{d}y \frac{1}{\sqrt{-x^2+2xy-y^2+k_\perp^2(x+y)-\frac{1}{4}k_\perp^4}} \\ & \times\theta\left[-x^2+2xy-y^2+k_\perp^2(x+y)-\frac{1}{4}k_\perp^4\right] \\ & \times\theta\left[-x^2+2xy-y^2-k_z^2(x+y)+2\omega(x-y)-\left(\frac{1}{2}k_z^2-\omega\right)^2+2\mu k_z^2\right]. \end{aligned} \tag{73}$$

The two step functions determine a region in the xy-plane that is enclosed between two parabolas. Integrating first over $x+y$, and subsequently over $x-y$, we arrive at the expression

$$S_{\mathrm{inc}}(\mathbf{k},\omega) = \frac{\mu}{2\pi k}\left[1-\frac{1}{2}\left(\frac{\omega}{\sqrt{\mu}k}-\frac{k}{2\sqrt{\mu}}\right)^2\right] \tag{74}$$

valid for $-\sqrt{2\mu}\,k+\frac{1}{2}k^2 \leqslant \omega \leqslant \sqrt{2\mu}\,k+\frac{1}{2}k^2$. The coherent part of the dynamic structure factor may be discussed along similar lines. One finds in the field-free limit

$$S_{\mathrm{coh}}(\mathbf{k},\omega) = -\frac{\mu}{2\pi k}\left[1-\frac{1}{2}\left(\frac{|\omega|}{\sqrt{\mu}k}+\frac{k}{2\sqrt{\mu}}\right)^2\right] \tag{75}$$

for $k \leqslant 2\sqrt{2\mu}$ and $|\omega| \leqslant \sqrt{2\mu}\,k-\frac{1}{2}k^2$. Expressions (74) and (75) agree with the well-known results for a degenerate fermion gas without a magnetic field [3, 6, 11].

7. Edge effects: hard wall perpendicular to the field

In the presence of a hard wall at $z = 0$ the energy eigenfunctions $\varphi_{n_y,n_z,n}(\mathbf{r})$ in the domain $z > 0$ are related to (14) by a reflection principle:

$$\varphi_{n_y,n_z,n}(\mathbf{r}) = \frac{1}{\sqrt{2}}[\psi_{n_y,n_z,n}(\mathbf{r}) - \psi_{n_y,n_z,n}(\tilde{\mathbf{r}})] \tag{76}$$

with $\tilde{\mathbf{r}} = (x, y, -z)$. Hence, both G_μ and G can be obtained immediately from the results of section 3. We will only consider the case $z = z'$. From (23) we get

$$G_\mu(\mathbf{r}, \mathbf{r}', t) = \frac{B^{3/2}}{4\pi^{3/2}\tau^{1/2}}\,\mathrm{e}^{-\frac{1}{4}(\rho_\perp-\rho'_\perp)^2+\frac{1}{2}\mathrm{i}(\xi+\xi')(\eta-\eta')}\sum_n{}' \mathrm{e}^{-\mathrm{i}(n+\frac{1}{2})\tau} L_n\left[\frac{1}{2}(\boldsymbol{\rho}_\perp - \boldsymbol{\rho}'_\perp)^2\right]$$
$$\times\left\{F\left[\sqrt{\tau\left(\nu - n - \frac{1}{2}\right)}, 0\right] - \mathrm{e}^{2\mathrm{i}\zeta^2/\tau} F\left[\sqrt{\tau\left(\nu - n - \frac{1}{2}\right)}, \sqrt{\frac{2}{\tau}}\zeta\right]\right\}. \tag{77}$$

Likewise, we find from (26):

$$G(\mathbf{r}, \mathbf{r}', t) = -\frac{(1+\mathrm{i})B^{3/2}}{8\pi^{3/2}\tau^{1/2}\sin\left(\frac{1}{2}\tau\right)}\left(1 - \mathrm{e}^{2\mathrm{i}\zeta^2/\tau}\right)\mathrm{e}^{\frac{1}{4}\mathrm{i}\cot(\frac{1}{2}\tau)(\rho_\perp-\rho'_\perp)^2+\frac{1}{2}\mathrm{i}(\xi+\xi')(\eta-\eta')}. \tag{78}$$

For equal-position vectors ($\mathbf{r} = \mathbf{r}'$) the Laguerre polynomial and the space-dependent exponential drop out from (77), so that one gets

$$G_\mu(\mathbf{r}, \mathbf{r}, t) = \frac{B^{3/2}}{4\pi^{3/2}\tau^{1/2}}\sum_n{}' \mathrm{e}^{-\mathrm{i}(n+\frac{1}{2})\tau}$$
$$\times\left\{F\left[\sqrt{\tau\left(\nu - n - \frac{1}{2}\right)}, 0\right] - \mathrm{e}^{2\mathrm{i}\zeta^2/\tau} F\left[\sqrt{\tau\left(\nu - n - \frac{1}{2}\right)}, \sqrt{\frac{2}{\tau}}\zeta\right]\right\}. \tag{79}$$

Expression (78) for G reduces in this case to

$$G(\mathbf{r}, \mathbf{r}, t) = -\frac{(1+\mathrm{i})B^{3/2}}{8\pi^{3/2}\tau^{1/2}\sin\left(\frac{1}{2}\tau\right)}\left(1 - \mathrm{e}^{2\mathrm{i}\zeta^2/\tau}\right). \tag{80}$$

The static correlation function G_μ ($t = 0$) follows upon using (27) in (77):

$$G_\mu(\mathbf{r}, \mathbf{r}', t = 0) = \frac{B^{3/2}}{2\pi^2}\,\mathrm{e}^{-\frac{1}{4}(\rho_\perp-\rho'_\perp)^2+\frac{1}{2}\mathrm{i}(\xi+\xi')(\eta-\eta')}\sum_n{}' L_n\left[\frac{1}{2}(\boldsymbol{\rho}_\perp - \boldsymbol{\rho}'_\perp)^2\right]$$
$$\times\sqrt{2\left(\nu - n - \frac{1}{2}\right)}\left\{1 - \frac{\sin\left[2\sqrt{2\left(\nu - n - \frac{1}{2}\right)}\zeta\right]}{2\sqrt{2\left(\nu - n - \frac{1}{2}\right)}\zeta}\right\}. \tag{81}$$

This expression may also be obtained directly from (28). For coinciding positions the Laguerre polynomial and the exponential prefactor again drop out. The function G reduces to $\delta(\mathbf{r} - \mathbf{r}')$ in the static limit, as in the bulk case.

Expressions (77) and (78) (or (79) and (80) for coinciding positions) for G_μ and G may be compared to their counterparts (23) and (26) (or (34) and (35)) for the bulk. The contributions from the reflection in the wall lead to a change in the dependence of the correlation function on time. This becomes manifest upon choosing positions near the wall. A Taylor expansion for small ζ leads to the following expressions for the correlation functions near the wall:

$$G_\mu(\mathbf{r},\mathbf{r}',t) \approx -\frac{\mathrm{i}\zeta^2 B^{3/2}}{\pi^{3/2}\tau^{3/2}}\,\mathrm{e}^{-\frac{1}{4}(\rho_\perp-\rho'_\perp)^2+\frac{1}{2}\,\mathrm{i}(\xi+\xi')(\eta-\eta')}$$
$$\times \sum_n{}' L_n\left[\frac{1}{2}(\rho_\perp-\rho'_\perp)^2\right]\left\{\mathrm{e}^{-\mathrm{i}(n+\frac{1}{2})\tau}\left[C\left(\sqrt{\tau\left(\nu-n-\frac{1}{2}\right)}\right)\right.\right.$$
$$\left.\left.-\,\mathrm{i}S\left(\sqrt{\tau\left(\nu-n-\frac{1}{2}\right)}\right)\right]-\sqrt{\frac{2}{\pi}}\,\mathrm{e}^{-\mathrm{i}\nu\tau}\sqrt{\tau\left(\nu-n-\frac{1}{2}\right)}\right\} \tag{82}$$

and

$$G(\mathbf{r},\mathbf{r}',t) \approx -\frac{(1-\mathrm{i})\zeta^2 B^{3/2}}{4\pi^{3/2}\tau^{3/2}\sin\left(\frac{1}{2}\tau\right)}\,\mathrm{e}^{\frac{1}{4}\mathrm{i}\cot(\frac{1}{2}\tau)(\rho_\perp-\rho'_\perp)^2+\frac{1}{2}\mathrm{i}(\xi+\xi')(\eta-\eta')}. \tag{83}$$

For equal positions these become

$$G_\mu(\mathbf{r},\mathbf{r},t) \approx -\frac{\mathrm{i}\zeta^2 B^{3/2}}{\pi^{3/2}\tau^{3/2}}\sum_n{}'\left\{\mathrm{e}^{-\mathrm{i}(n+\frac{1}{2})\tau}\left[C\left(\sqrt{\tau\left(\nu-n-\frac{1}{2}\right)}\right)\right.\right.$$
$$\left.\left.-\,\mathrm{i}S\left(\sqrt{\tau\big(\nu-n-\frac{1}{2}\big)}\right)\right]-\sqrt{\frac{2}{\pi}}\,\mathrm{e}^{-\mathrm{i}\nu\tau}\sqrt{\tau\big(\nu-n-\frac{1}{2}\big)}\right\} \tag{84}$$

and

$$G(\mathbf{r},\mathbf{r},t) \approx -\frac{(1-\mathrm{i})\zeta^2 B^{3/2}}{4\pi^{3/2}\tau^{3/2}\sin\left(\frac{1}{2}\tau\right)}. \tag{85}$$

For large τ the Fresnel integrals between the curly brackets in (82) and (84) can be neglected, so that both expressions show an algebraic decay proportional to τ^{-1}. From (82) we get in this way

$$G_\mu(\mathbf{r},\mathbf{r}',t) \approx \frac{\mathrm{i}\zeta^2 B^{3/2}}{\pi^2\tau}\,\mathrm{e}^{-\frac{1}{4}(\rho_\perp-\rho'_\perp)^2+\frac{1}{2}\mathrm{i}(\xi+\xi')(\eta-\eta')-\mathrm{i}\nu\tau}\sum_n{}' L_n\left[\frac{1}{2}(\rho_\perp-\rho'_\perp)^2\right]\sqrt{2\left(\nu-n-\frac{1}{2}\right)} \tag{86}$$

which for coinciding positions reduces to

$$G_\mu(\mathbf{r},\mathbf{r},t) \approx \frac{\mathrm{i}\zeta^2 B^{3/2}}{\pi^2\tau}\,\mathrm{e}^{-\mathrm{i}\nu\tau}\sum_n{}'\sqrt{2\left(\nu-n-\frac{1}{2}\right)} = \frac{2\mathrm{i}\zeta^2}{\tau}\,\mathrm{e}^{-\mathrm{i}\nu\tau}n_\infty \tag{87}$$

with n_∞ being the bulk particle density.

Comparing (87) with (36), one concludes that the behaviour of the correlation function G_μ, and hence of the coherent structure function S_{coh}, has changed owing to the presence of the wall. In fact, whereas in the bulk the coherent structure function for coinciding positions decays proportional to $1/\tau$ for large time (with a periodic modulation), the decay near the wall is faster, namely proportional to τ^{-2}. The function G has changed as well, with a decay proportional to $\tau^{-3/2}$ near the wall, which leads to a corresponding change in the behaviour of S_{inc}. However, the recurring singularities at $\tau = 2\pi m$ dominate the picture completely in this case, and these are not altered by the wall.

In figure 4 the normalized coherent structure function $S_{\mathrm{coh}}(\mathbf{r},\mathbf{r}',t)/S_{\mathrm{coh}}(\mathbf{r},\mathbf{r}',0)$ is shown for several values of the distance from the wall ζ and the position difference $|\xi-\xi'|$. The

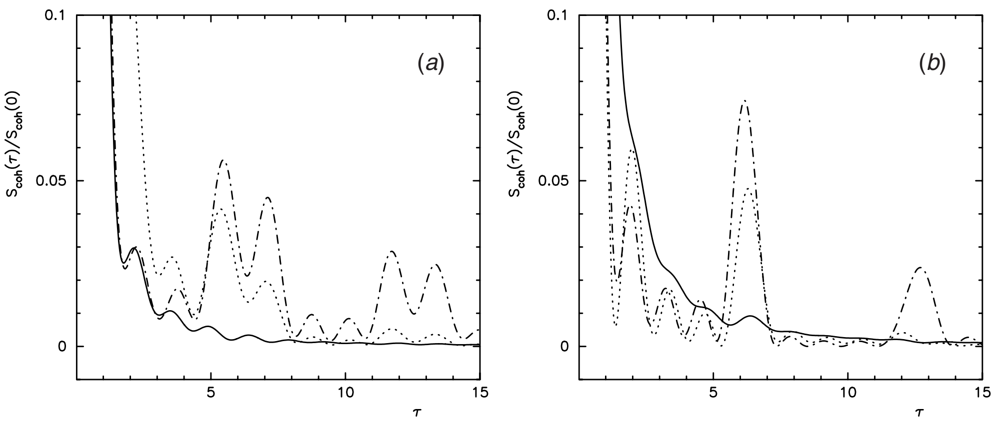

Figure 4. Coherent structure function as a function of time τ, at $\nu = 5$, for position difference $|\xi - \xi'| = 1$ (*a*), and for coinciding positions (*b*). The curves, which start at 1 for $\tau = 0$, give the normalized structure function for distances ζ from the wall equal to 0.1 (——), 1.5 (······) and ∞ (bulk, — · —).

faster time decay of the structure function for positions near the wall is manifest. Revivals are (almost) completely suppressed in this case.

8. Edge effects: hard wall parallel to the field

Let us now consider wall effects in a different geometric arrangement, namely for a wall parallel to the field. For a hard wall at $x = 0$ the energy eigenfunctions $\varphi_{n_y,n_z,n}(\mathbf{r})$ for $x > 0$ can be expressed in parabolic cylinder functions [2, 9] as

$$\varphi_{n_y,n_z,n}(\mathbf{r}) = \frac{B^{1/4}}{L} \frac{D_{\varepsilon_n(\kappa_y)-1/2}[\sqrt{2}(\xi - \kappa_y)]}{\left\{\int_0^\infty \mathrm{d}\xi' D^2_{\varepsilon_n(\kappa_y)-1/2}[\sqrt{2}(\xi' - \kappa_y)]\right\}^{1/2}} \, \mathrm{e}^{\mathrm{i}\kappa_y\eta + \mathrm{i}\kappa_z\zeta}. \tag{88}$$

As for the bulk case, the wavevectors are given as $\kappa_i = 2\pi n_i/(L\sqrt{B})$, with integer n_i for $i = y, z$. Furthermore, the function $\varepsilon_n(\kappa)$ occurring in the index of the cylinder functions is determined by [2]

$$D_{\varepsilon_n(\kappa)-1/2}(-\sqrt{2}\kappa) = 0. \tag{89}$$

The same function shows up in the eigenvalue associated with (88)

$$E_{n_y,n_z,n} = B\varepsilon_n(\kappa_y) + \tfrac{1}{2}k_z^2. \tag{90}$$

The time correlation function G_μ for the degenerate case follows by inserting (88) in (7). We will only consider the case $x = x'$. Using (18) and (21), with $n + \frac{1}{2}$ replaced by $\varepsilon_n(\kappa_y)$, we get

$$G_\mu(\mathbf{r}, \mathbf{r}', t) = \frac{B^{3/2}}{4\pi^{3/2}\tau^{1/2}} \, \mathrm{e}^{\frac{1}{2}\mathrm{i}(\zeta-\zeta')^2/\tau} {\sum_n}' \int_{\kappa_n(\nu)}^\infty \mathrm{d}\kappa_y \, \mathrm{e}^{\mathrm{i}\kappa_y(\eta-\eta')-\mathrm{i}\varepsilon_n(\kappa_y)\tau}$$
$$\times \frac{D^2_{\varepsilon_n(\kappa_y)-1/2}[\sqrt{2}(\xi - \kappa_y)]}{\int_0^\infty \mathrm{d}\xi'' D^2_{\varepsilon_n(\kappa_y)-1/2}[\sqrt{2}(\xi'' - \kappa_y)]} F\left(\sqrt{\tau[\nu - \varepsilon_n(\kappa_y)]}, \frac{\zeta - \zeta'}{\sqrt{2\tau}}\right) \tag{91}$$

where $\kappa_n(\nu)$ is defined by the condition $\varepsilon_n[\kappa_n(\nu)] = \nu$ with $n \leqslant \nu - \frac{1}{2}$. In an analogous way, the function G for $x = x'$ is found as

$$G(\mathbf{r}, \mathbf{r}', t) = \frac{(1-\mathrm{i})B^{3/2}}{4\pi^{3/2}\tau^{1/2}}\, \mathrm{e}^{\frac{1}{2}\mathrm{i}(\zeta-\zeta')^2/\tau} \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} \mathrm{d}\kappa_y\, \mathrm{e}^{\mathrm{i}\kappa_y(\eta-\eta')-\mathrm{i}\varepsilon_n(\kappa_y)\tau} \times \frac{D^2_{\varepsilon_n(\kappa_y)-1/2}[\sqrt{2}(\xi-\kappa_y)]}{\int_0^\infty \mathrm{d}\xi''\, D^2_{\varepsilon_n(\kappa_y)-1/2}[\sqrt{2}(\xi''-\kappa_y)]}. \tag{92}$$

Here, the sum extends over all non-negative integers n and the integral over all κ_y, in contrast to the sum and integral in (91).

For large values of x expressions (91) and (92) reduce to their bulk forms (23) and (24). In fact, for $\xi \to \infty$ only large values of κ_y contribute to the integrals. For these large κ_y the function $\varepsilon_n(\kappa_y)$ reduces to $n + \frac{1}{2}$. Furthermore, the parabolic cylinder function $D_n(\sqrt{2}u)$ for integer n can be rewritten in terms of a Hermite polynomial as $2^{-n/2}\,\mathrm{e}^{-u^2/2} H_n(u)$. Finally, the lower bounds of both the integrals over ξ'' in the denominators in (91), (92) and of the integral over κ_y in (91) can be extended to $-\infty$. Evaluating the integrals with the use of (19), one recovers (23) and (24).

As a special case we consider the time correlation functions G_μ and G for equal position vectors, as in sections 3 and 7. One finds from (91) and (92)

$$G_\mu(\mathbf{r}, \mathbf{r}, t) = \frac{B^{3/2}}{2\pi^{3/2}\tau^{1/2}} {\sum_n}' \int_{\kappa_n(\nu)}^{\infty} \mathrm{d}\kappa_y\, \mathrm{e}^{-\mathrm{i}\varepsilon_n(\kappa_y)\tau} \frac{D^2_{\varepsilon_n(\kappa_y)-1/2}[\sqrt{2}(\xi-\kappa_y)]}{\int_0^\infty \mathrm{d}\xi''\, D^2_{\varepsilon_n(\kappa_y)-1/2}[\sqrt{2}(\xi''-\kappa_y)]} \times \left\{C\left(\sqrt{\tau[\nu-\varepsilon_n(\kappa_y)]}\right) - \mathrm{i}S(\sqrt{\tau[\nu-\varepsilon_n(\kappa_y)]})\right\} \tag{93}$$

and

$$G(\mathbf{r}, \mathbf{r}, t) = \frac{(1-\mathrm{i})B^{3/2}}{4\pi^{3/2}\tau^{1/2}} \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} \mathrm{d}\kappa_y\, \mathrm{e}^{-\mathrm{i}\varepsilon_n(\kappa_y)\tau} \frac{D^2_{\varepsilon_n(\kappa_y)-1/2}[\sqrt{2}(\xi-\kappa_y)]}{\int_0^\infty \mathrm{d}\xi''\, D^2_{\varepsilon_n(\kappa_y)-1/2}[\sqrt{2}(\xi''-\kappa_y)]}. \tag{94}$$

These expressions are the counterparts of (34), (35) for the bulk, and of (79), (80) for the other geometry. For large t the Fresnel integrals in (93) can be replaced by their asymptotic value $\frac{1}{2}$.

In figure 5 numerical results for the normalized coherent structure function, which follows by inserting (91) or (93) in (4), are presented. Curves are drawn for several values of the distance ξ from the wall, and for various position differences, with a few different orientations of the latter. The general picture of these figures is the same as that obtained for the other geometry: near the wall the time decay of the structure function is considerably faster than in the bulk. More details on these figures will be given below, when discussing the time correlation functions near the wall.

For $t = 0$ the correlation function G_μ can be simplified by using the asymptotic relation (27). One gets from (91)

$$G_\mu(\mathbf{r}, \mathbf{r}', t=0) = \frac{B^{3/2}}{2\pi^2|\zeta-\zeta'|} {\sum_n}' \int_{\kappa_n(\nu)}^{\infty} \mathrm{d}\kappa_y\, \mathrm{e}^{\mathrm{i}\kappa_y(\eta-\eta')} \times \frac{D^2_{\varepsilon_n(\kappa_y)-1/2}[\sqrt{2}(\xi-\kappa_y)]}{\int_0^\infty \mathrm{d}\xi''\, D^2_{\varepsilon_n(\kappa_y)-1/2}[\sqrt{2}(\xi''-\kappa_y)]} \sin\left(\sqrt{2[\nu-\varepsilon_n(\kappa_y)]}|\zeta-\zeta'|\right) \tag{95}$$

which agrees with a result from [2], if spin degeneracy is taken into account. As before, the second correlation function $G(\mathbf{r}, \mathbf{r}', t)$ becomes a spatial delta function in the static limit.

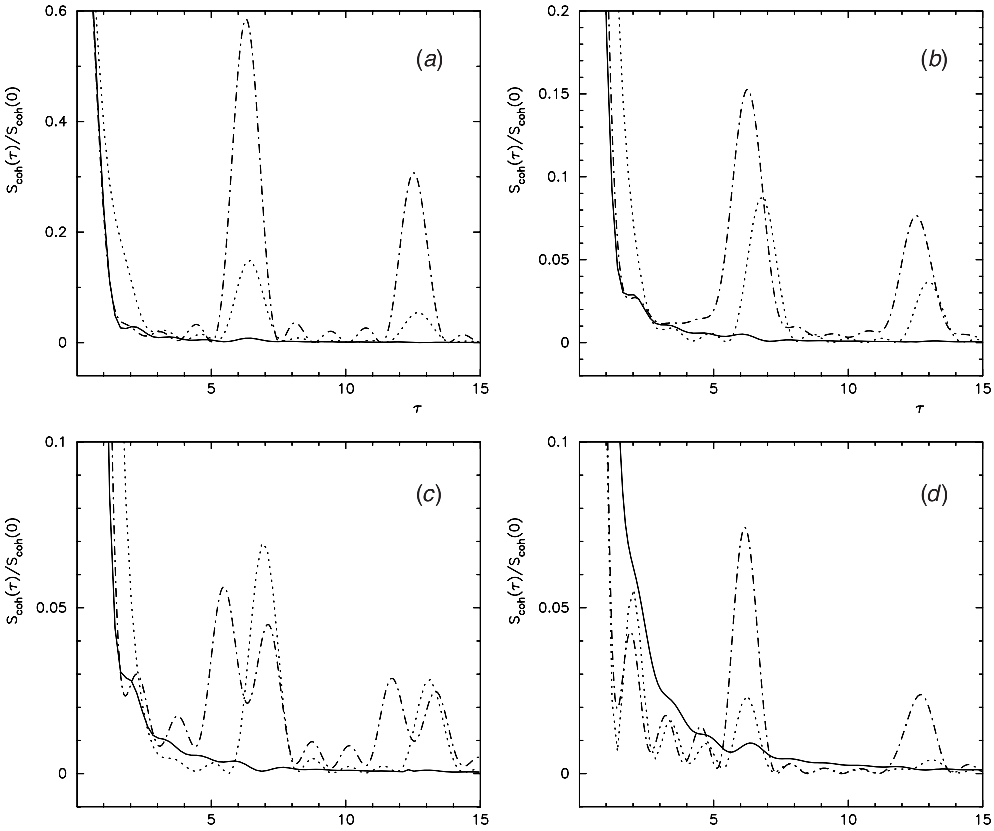

Figure 5. Coherent structure function as a function of time τ, at $\nu = 5$, for position difference $|\rho - \rho'| = 1$ (*a*)–(*c*), and for coinciding positions (*d*). The first three correspond to θ equal to 0 (*a*), $\pi/4$ (*b*), $\pi/2$ (*c*), with $\theta = \arctan(|\eta - \eta'|/|\zeta - \zeta'|)$. The curves, which start at 1 for $\tau = 0$, give the normalized structure function for distances ξ from the wall equal to 0.1 (——), 1.5 (· · · · · ·) and ∞ (bulk, — · —).

As in the previous section, we want to study the behaviour of the time correlation functions in the vicinity of the wall. To that end we start from (91) and (92), and use the identity [2]

$$\frac{1}{\int_0^\infty \mathrm{d}\xi'' D^2_{\varepsilon_n(\kappa)-1/2}[\sqrt{2}(\xi'' - \kappa)]} = -\frac{1}{2\pi}\Gamma^2\left[-\varepsilon_n(\kappa) + \frac{1}{2}\right] D^2_{\varepsilon_n(\kappa)-1/2}(\sqrt{2}\kappa)\frac{\mathrm{d}\varepsilon_n(\kappa)}{\mathrm{d}\kappa}. \tag{96}$$

Furthermore, we make a Taylor expansion of the parabolic cylinder function in the numerators of (91) and (92). Subsequently, we employ the Wronskian identity [9]

$$D_{\varepsilon_n(\kappa)-1/2}(\sqrt{2}\kappa)\frac{\partial}{\partial\kappa}D_\nu(-\sqrt{2}\kappa)\bigg|_{\nu=\varepsilon_n(\kappa)-1/2} = \frac{2\sqrt{\pi}}{\Gamma\left[-\varepsilon_n(\kappa) + \frac{1}{2}\right]} \tag{97}$$

which holds as a consequence of (89). Upon changing the integration variable from κ_y to ε in (91), we finally obtain the time correlation function for positions near the wall as

$$G_\mu(\mathbf{r}, \mathbf{r}', t) \approx \frac{\xi^2 B^{3/2}}{2\pi^{3/2}\tau^{1/2}}\, \mathrm{e}^{\frac{1}{2}\mathrm{i}(\zeta-\zeta')^2/\tau} {\sum_n}' \int_{n+\frac{1}{2}}^{\nu} \mathrm{d}\varepsilon\, \mathrm{e}^{\mathrm{i}\kappa_n(\varepsilon)(\eta-\eta')-\mathrm{i}\varepsilon\tau} F\left[\sqrt{\tau(\nu-\varepsilon)}, \frac{\zeta-\zeta'}{\sqrt{2\tau}}\right]. \tag{98}$$

Likewise, (92) gives

$$G(\mathbf{r}, \mathbf{r}', t) \approx \frac{(1-\mathrm{i})\xi^2 B^{3/2}}{2\pi^{3/2}\tau^{1/2}} \, \mathrm{e}^{\frac{1}{2}\mathrm{i}(\zeta-\zeta')^2/\tau} \sum_{n=0}^{\infty} \int_{n+\frac{1}{2}}^{\infty} \mathrm{d}\varepsilon \, \mathrm{e}^{\mathrm{i}\kappa_n(\varepsilon)(\eta-\eta')-\mathrm{i}\varepsilon\tau}. \tag{99}$$

These expressions may be compared to (82) and (83) for the other geometry. We will now consider the time correlation functions in the vicinity of the wall for some particular orientations of the position difference, and for the case of coinciding positions.

8.1. Longitudinal position difference ($y = y'$, $z \neq z'$)

Putting $\eta = \eta'$ in expression (98) for G_μ, we can evaluate the integral by a partial integration. The result is

$$G_\mu(\mathbf{r}, \mathbf{r}', t) \approx -\frac{\mathrm{i}\xi^2 B^{3/2}}{2\pi^{3/2}\tau^{3/2}} \, \mathrm{e}^{\frac{1}{2}\mathrm{i}(\zeta-\zeta')^2/\tau} {\sum_n}' \left\{ \mathrm{e}^{-\mathrm{i}(n+\frac{1}{2})\tau} F\left[\sqrt{\tau\left(\nu - n - \frac{1}{2}\right)}, \frac{\zeta-\zeta'}{\sqrt{2\tau}}\right] \right.$$
$$\left. - 2\sqrt{\frac{\tau}{\pi}} \, \mathrm{e}^{-\mathrm{i}\nu\tau - \frac{1}{2}\mathrm{i}(\zeta-\zeta')^2/\tau} \frac{\sin\left[\sqrt{2\left(\nu - n - \frac{1}{2}\right)}|\zeta-\zeta'|\right]}{|\zeta-\zeta'|} \right\}. \tag{100}$$

For large separations $|\zeta - \zeta'|$ we obtain with the use of (27):

$$G_\mu(\mathbf{r}, \mathbf{r}', t) \approx -\frac{\xi^2 B^{3/2}}{\pi^2|\zeta-\zeta'|^2} \, \mathrm{e}^{-\mathrm{i}\nu\tau} {\sum_n}' \sqrt{2\left(\nu - n - \frac{1}{2}\right)} \cos\left[\sqrt{2\left(\nu - n - \frac{1}{2}\right)}|\zeta-\zeta'|\right]. \tag{101}$$

For $t = 0$ expressions (100) and (101) reduce to static results found before [2]. For large τ the second term between the curly brackets in (100) dominates, so that one gets

$$G_\mu(\mathbf{r}, \mathbf{r}', t) \approx \frac{\mathrm{i}\xi^2 B^{3/2}}{\pi^2\tau} \, \mathrm{e}^{-\mathrm{i}\nu\tau} {\sum_n}' \frac{\sin\left[\sqrt{2\left(\nu - n - \frac{1}{2}\right)}|\zeta-\zeta'|\right]}{|\zeta-\zeta'|}. \tag{102}$$

Upon comparing this result with (86), which holds for the other geometry, one finds a similar behaviour as a function of time: both have a decay proportional to τ^{-1}, which differs from the $\tau^{-1/2}$ decay of G_μ for the bulk. These changes in the behaviour of G_μ entail modifications in the decay properties of the coherent structure function that have been discussed in the previous section. The differences in the spatial dependence between (86) and (102) are analogous to those found in the bulk static correlation function (28) for transverse and longitudinal directions of the position difference, respectively.

Numerical results for the coherent structure function with a purely longitudinal position difference are given in figure 5(*a*). For a small distance from the wall ($\xi = 0.1$) the structure function decays rapidly. It does not show the revivals around $\tau = 2\pi m$, which are a prominent feature of the bulk structure function, and which we have seen already in figure 1. For intermediate distances ($\xi = 1.5$ in the figure) these revivals have lost part of their strength.

As for G, the integral in (99) becomes trivial for $\eta = \eta'$, so that one gets

$$G(\mathbf{r}, \mathbf{r}', t) \approx -\frac{(1-\mathrm{i})\xi^2 B^{3/2}}{4\pi^{3/2}\tau^{3/2}\sin\left(\frac{1}{2}\tau\right)} \, \mathrm{e}^{\frac{1}{2}\mathrm{i}(\zeta-\zeta')^2/\tau} \tag{103}$$

which may be compared to (83) for the other geometry, and to (33) for the bulk. As before, the time dependence shows the usual characteristic recurring singularity and a faster decay than in the bulk.

8.2. Transverse position difference ($z = z'$, $y \neq y'$)

In this case, expression (98) for G_μ becomes

$$G_\mu(\mathbf{r}, \mathbf{r}', t) \approx \frac{\xi^2 B^{3/2}}{\pi^{3/2}\tau^{1/2}} {\sum_n}' \int_{n+\frac{1}{2}}^{\nu} \mathrm{d}\varepsilon\, \mathrm{e}^{\mathrm{i}\kappa_n(\varepsilon)(\eta-\eta')-\mathrm{i}\varepsilon\tau} \left\{C\left[\sqrt{\tau(\nu-\varepsilon)}\right] - \mathrm{i}S\left[\sqrt{\tau(\nu-\varepsilon)}\right]\right\}. \tag{104}$$

In general, the integral cannot be simplified further. However, for large $|\eta - \eta'|$ one may find its asymptotic behaviour, as it is determined by the upper boundary. One gets

$$G_\mu(\mathbf{r}, \mathbf{r}', t) \approx \frac{\xi^2 B^{3/2}}{\sqrt{2}\pi^{3/2}} \mathrm{e}^{-\mathrm{i}\nu\tau} \frac{\mathrm{e}^{\frac{3}{4}\pi\mathrm{i}\,\mathrm{sgn}(\eta-\eta')}}{|\eta-\eta'|^{3/2}} {\sum_n}' \mathrm{e}^{\mathrm{i}\kappa_n(\nu)(\eta-\eta')} \left[-\frac{\mathrm{d}\kappa_n(\nu)}{\mathrm{d}\nu}\right]^{-3/2} \tag{105}$$

which for $t = 0$ reduces to an earlier result [2]. As in (101) the time dependence is given by a trivial phase factor. The space dependence is quite different from that of (101).

For G as given by (99), the choice $\zeta = \zeta'$ produces only a slight simplification. The integral in (99) remains as it stands; it is difficult to evaluate, even for large $|\eta - \eta'|$.

In figure 5 (*c*) some numerical results for the coherent structure function with a purely transverse position difference are presented. As before, the structure function decays rapidly, if the distance from the wall is small ($\xi = 0.1$). The behaviour is rather similar to that for the other geometry (see figure 4(*a*)). At intermediate distances ($\xi = 1.5$) some revivals are present, as in the longitudinal case (see figure 5(*a*)), albeit with a smaller amplitude (note the difference in scale). At this distance from the wall the structure function is quite different for the two geometries, as can be seen by comparing with the corresponding curve in figure 4(*a*). In the bulk the revivals have a more complicated structure, as we have seen already in figures 1 and 4. The intermediate case of a position difference that is oriented at an angle $\pi/4$ with respect to the field leads to a behaviour of the coherent structure function that interpolates between the purely longitudinal and the purely transverse cases, as can be seen in figure 5(*b*).

8.3. Coinciding positions ($y = y'$, $z = z'$)

For coinciding positions we get from (100)

$$G_\mu(\mathbf{r}, \mathbf{r}, t) \approx -\frac{\mathrm{i}\xi^2 B^{3/2}}{\pi^{3/2}\tau^{3/2}} {\sum_n}' \left\{ \mathrm{e}^{-\mathrm{i}(n+\frac{1}{2})\tau} \left[C\left(\sqrt{\tau\left(\nu - n - \frac{1}{2}\right)}\right) - \mathrm{i}S\left(\sqrt{\tau\left(\nu - n - \frac{1}{2}\right)}\right)\right] - \sqrt{\frac{2}{\pi}}\,\mathrm{e}^{-\mathrm{i}\nu\tau}\sqrt{\tau\left(\nu - n - \frac{1}{2}\right)} \right\}. \tag{106}$$

Remarkably enough, this expression is identical to (84), apart from the interchange of ξ and ζ. Hence, near a wall the time correlation function for equal positions has a universal form, independent of the orientation of the wall with respect to the field.

The same conclusion is reached for G. Indeed, from (103) we find for $\zeta = \zeta'$:

$$G(\mathbf{r}, \mathbf{r}, t) \approx -\frac{(1-\mathrm{i})\xi^2 B^{3/2}}{4\pi^{3/2}\tau^{3/2}\sin\left(\frac{1}{2}\tau\right)} \tag{107}$$

which has the same form as (85).

For large τ the second term between the curly brackets in (106) dominates. Introducing the bulk particle density, we may write G_μ for coinciding positions near the wall in this asymptotic time regime in a form analogous to (87):

$$G_\mu(\mathbf{r}, \mathbf{r}, t) \approx \frac{2\mathrm{i}\xi^2}{\tau}\,\mathrm{e}^{-\mathrm{i}\nu\tau} n_\infty. \tag{108}$$

In accordance with the more general result discussed below (106), the long-time asymptotic behaviour of the equal-position time correlation function near a wall is independent of the orientation of the wall with respect to the field. The asymptotic form has an even stronger universality property: it is independent of the strength of the field as well. This follows by reintroducing the original (non-scaled) space and time variables, and the chemical potential μ. In terms of these variables (108) reads

$$G_\mu(\mathbf{r}, \mathbf{r}, t) \approx \frac{2\mathrm{i}x^2}{t}\,\mathrm{e}^{-\mathrm{i}\mu t} n_\infty. \tag{109}$$

The right-hand side is indeed independent of the magnetic field (apart from the implicit dependence through the bulk particle density). It should be noted that in the bulk the long-time tail in G_μ for equal positions does depend on the magnetic field, as we have seen in (36).

The universality property found for G_μ does not hold for G. In fact, reintroducing the original variables in (107) we get

$$G(\mathbf{r}, \mathbf{r}, t) \approx -\frac{(1-\mathrm{i})x^2 B}{4\pi^{3/2} t^{3/2} \sin\left(\frac{1}{2}Bt\right)}. \tag{110}$$

Clearly, this result depends on the magnetic field strength. It shows the same characteristic recurrent pole structure as we have found in the bulk (see (35)). As in the bulk, the poles at $t = 2\pi n/B$ move when the field strength changes.

Numerical results for the coherent structure function with coinciding positions are given in figure 5(*d*). In the vicinity of the wall ($\xi = 0.1$) the structure function is (almost) identical to that near a wall perpendicular to the field (see the curve for $\zeta = 0.1$ in figure 4(*b*)), in accordance with the analytical results (84) and (106). For intermediate distances from the wall the curves for the two geometries are rather different, while they agree again in the bulk, of course.

9. Concluding remarks

In this paper, we have studied time-dependent pair correlations in a quantum gas of charged free fermions in a uniform external field. These correlations can be described in terms of two correlation functions $G_{\mu,T}(\mathbf{r}, \mathbf{r}', t)$ and $G(\mathbf{r}, \mathbf{r}', t)$, which together determine both the coherent and the incoherent parts of the time-dependent structure function $S(\mathbf{r}, \mathbf{r}', t)$, according to (3)–(6). Our main interest has been focused on the behaviour of these correlation functions for the completely degenerate quantum gas at temperature $T = 0$.

Owing to the presence of the magnetic field, the particle motion is quantized in terms of Landau levels with an associated cyclotron frequency. One of the aims of the present paper has been to show how the correlation functions are influenced by this cyclotron movement. Results (23) and (24) express the functions $G_{\mu,T=0}$ and G as sums over Landau levels, involving Laguerre polynomials for the spatial dependence and Fresnel integrals for the time dependence. Their time behaviour is sensitive to the orientation of the position difference with respect to the magnetic field, as is demonstrated in (29)–(33). The time behaviour of the ensuing coherent part of the structure function, as shown in figure 1, is characterized by recurrent 'revivals', with a markedly increased strength of the correlation effects at values of the time t equal to a multiple of the inverse cyclotron frequency. These revivals are particularly clear for position differences in the longitudinal direction. The incoherent part of the structure factor is influenced even more strongly: it is singular for the values of t mentioned above, as can be seen from (26).

The time dependence of the correlation functions and the structure function can be translated to a frequency dependence. Passing via the intermediate scattering function $S(\mathbf{k}, t)$, for which we have derived expressions (44) and (54), we arrive at the dynamical structure factor $S(\mathbf{k}, \omega)$, as given by (60) and (62). The curves in figures 2 and 3 show how both the coherent and the incoherent parts of $S(\mathbf{k}, \omega)$ get a spike structure for purely transverse orientations of the wavevector. The gradual change of these curves as the wavevector gets a longitudinal component can be inferred from the figures. For a purely longitudinal wavevector a step structure is found, in accordance with (64).

The above results pertain to the bulk of magnetized quantum gases. As is well known, these systems show additional interesting physical phenomena near the walls of the containers to which they are confined. A second goal of the present paper has been to investigate how the time-dependent correlations change in the vicinity of a confining wall. In general, these edge effects depend on the orientation of the wall with respect to the field. They have been determined for two specific orientations, namely for a wall perpendicular to the field and for a wall parallel to the field. In the former case the cylinder symmetry of the field is not changed by the wall. As a consequence, the time correlation functions are easily obtained from those of the bulk, with results given in (77) and (78). The curves of figure 4 show that the main influence of the wall is a faster time decay of the correlations. For a wall parallel to the field it is a lot more cumbersome to determine the correlation functions, as the symmetry of the problem is now lost completely. The general results for the correlation functions are given in (91) and (92). The curves for the coherent structure function, which are presented in figure 5, show that once again the correlations die out faster than in the bulk. In particular, the characteristic revivals at multiples of the inverse cyclotron frequency, which are a conspicuous feature of the bulk coherent structure function for longitudinal position differences, are almost absent for positions near the wall. It appears that the wall perturbs the regular patterns of the bulk motion so strongly that the recurrences associated with the cyclotron frequency do no longer influence the correlation functions.

Appendix

In this appendix we consider several integral relations that are employed in the main text. The first identity, which has been used in section 4, is

$$\int_{-\infty}^{\infty} \mathrm{d}z\, \mathrm{e}^{-\mathrm{i}kz} F^*(a, zc) F(b, zc)$$
$$= -\frac{2\mathrm{i}}{k}\, \mathrm{e}^{\frac{1}{4}\mathrm{i}k^2/c^2} \{\mathrm{e}^{\mathrm{i}kb/c}\theta[4a^2c^2 - (k+2bc)^2] - \mathrm{e}^{-\mathrm{i}kb/c}\theta[4a^2c^2 - (k-2bc)^2]\}$$
$$+ \frac{2\mathrm{i}}{k}\, \mathrm{e}^{-\frac{1}{4}\mathrm{i}k^2/c^2} \{\mathrm{e}^{-\mathrm{i}ka/c}\theta[4b^2c^2 - (k+2ac)^2] - \mathrm{e}^{\mathrm{i}ka/c}\theta[4b^2c^2 - (k-2ac)^2]\} \tag{A.1}$$

with the function F as defined in (22) and with positive a, b, c and real $k \neq 0$. To prove it, one starts by using a partial integration to rewrite the left-hand side as

$$-\frac{\mathrm{i}\sqrt{2}c}{\sqrt{\pi}k} \int_{-\infty}^{\infty} \mathrm{d}z\, \mathrm{e}^{-\mathrm{i}kz} \big\{\big[\mathrm{e}^{\mathrm{i}(a+zc)^2} - \mathrm{e}^{\mathrm{i}(a-zc)^2}\big] F(b, zc) + \big[\mathrm{e}^{-\mathrm{i}(b+zc)^2} - \mathrm{e}^{-\mathrm{i}(b-zc)^2}\big] F^*(a, zc)\big\}. \tag{A.2}$$

To determine the integral on the right-hand side, we have to evaluate integrals of the form

$$G(k, a, b) = \int_{-\infty}^{\infty} \mathrm{d}z\, \mathrm{e}^{-\mathrm{i}kz+\mathrm{i}b^2z^2} F(a, zb) \tag{A.3}$$

for positive a, b and real k. We differentiate this function with respect to k. By adding a suitable multiple of G itself we arrive at

$$-2b^2\frac{\partial G(k,a,b)}{\partial k} - \mathrm{i}k\, G(k,a,b) = \int_{-\infty}^{\infty} \mathrm{d}\left(\mathrm{e}^{-\mathrm{i}kz+\mathrm{i}b^2z^2}\right) F(a,zb). \tag{A.4}$$

Integrating by parts and evaluating the ensuing integral the right-hand side becomes a combination of delta functions:

$$-2^{3/2}\sqrt{\pi}\, b\, \mathrm{e}^{-\mathrm{i}a^2}[\delta(k+2ab) - \delta(k-2ab)]. \tag{A.5}$$

By combining (A.4) and (A.5) we arrive at a differential equation, which can easily be solved as

$$G(k,a,b) = \frac{\sqrt{2\pi}}{b}\, \mathrm{e}^{-\frac{1}{4}\mathrm{i}k^2/b^2}\theta(4a^2b^2-k^2) + \bar{G}(a,b)\, \mathrm{e}^{-\frac{1}{4}\mathrm{i}k^2/b^2} \tag{A.6}$$

with an as yet unknown function $\bar{G}(a,b)$. To determine it we put $k = 0$, and differentiate both sides with respect to a. Using definition (A.3) we find, by differentiating $F(a, zb)$ and performing the resulting integral, that $G(0, a, b)$ is independent of a for all positive a and b. As a consequence $\bar{G}(a, b)$ is independent of a as well. Finally, by considering the limit $a \to \infty$ (for $k = 0$ as before) in (A.3) and employing (27), we prove $\lim_{a\to\infty} G(0, a, b) = \sqrt{2\pi}/b$, and hence $\lim_{a\to\infty} \bar{G}(a, b) = 0$. From these results we conclude that the term with $\bar{G}$ in (A.6) can be omitted, so that $G(a, b)$ is determined. After these preparations it is an easy matter to demonstrate the validity of (A.1).

A second identity (used in section 6) is

$$\int_0^a \mathrm{d}v\, \mathrm{e}^{-\mathrm{i}vt} J_0(\sqrt{v}b) F\left(\sqrt{(a-v)t}, \frac{c}{2\sqrt{t}}\right) = -\frac{\mathrm{i}}{t}\, \mathrm{e}^{\frac{1}{4}\mathrm{i}b^2/t} F\left(\sqrt{at}, \frac{\sqrt{b^2+c^2}}{2\sqrt{t}}\right) + \frac{2^{3/2}\mathrm{i}}{\sqrt{\pi t}\sqrt{b^2+c^2}}\, \mathrm{e}^{-\mathrm{i}at-\frac{1}{4}\mathrm{i}c^2/t} \sin\left[\sqrt{a(b^2+c^2)}\right] \tag{A.7}$$

for non-negative b, c (with $\sqrt{b^2+c^2} > 0$) and positive a, t. Before starting with the proof of this identity, we remark that its dependence on a is trivial, as follows from a rescaling of the variables. Hence, we will put $a = 1$ in the following. The function F in the integrand at the left-hand side of (A.7) may be replaced by the integral representation

$$F\left(\sqrt{(1-v)t}, \frac{c}{2\sqrt{t}}\right) = \sqrt{\frac{2t}{\pi}}\, \mathrm{e}^{-\frac{1}{4}\mathrm{i}c^2/t} \int_{-\sqrt{1-v}}^{\sqrt{1-v}} \mathrm{d}k\, \mathrm{e}^{-\mathrm{i}ck-\mathrm{i}tk^2} \tag{A.8}$$

as may be established from the definition of the Fresnel integrals (or by comparing (18) and (21)). Inserting the integral representation in (A.7), we arrive at a double integral over v and k. Introducing new integration variables ρ and θ by writing $v = \rho^2 \sin^2\theta$ and $k = \rho\cos\theta$, we rewrite this double integral as

$$2\int_0^1 \mathrm{d}\rho\, \rho^2\, \mathrm{e}^{-\mathrm{i}t\rho^2} \int_0^\pi \mathrm{d}\theta\, \sin\theta\, J_0(b\rho\sin\theta)\, \mathrm{e}^{-\mathrm{i}c\rho\cos\theta}. \tag{A.9}$$

The integral over θ turns out to depend on b and c only through the combination $\sqrt{b^2+c^2}$. This can be seen by using the standard representation for the Bessel function and writing the θ integral as

$$\frac{1}{2\pi}\int_0^\pi \mathrm{d}\theta \int_0^{2\pi} \mathrm{d}\varphi\, \sin\theta\, \mathrm{e}^{-\mathrm{i}b\rho\sin\theta\cos\varphi-\mathrm{i}c\rho\cos\theta}. \tag{A.10}$$

The argument of the exponential function is proportional to the scalar product of a unit vector in the direction of (θ, φ) and a constant vector with Cartesian components $(b, 0, c)$. Since the integration extends over all directions of the unit vector, the integral (A.10), and hence (A.9) as well, indeed depends on b and c through the length $\sqrt{b^2+c^2}$ only. Once this is known, we may evaluate (A.9) by choosing $b=0$ and replacing c by $\sqrt{b^2+c^2}$ in the answer. Since the Bessel function equals 1 for $b=0$, the integral (A.9) can easily be evaluated in terms of Fresnel integrals, with the result:

$$\frac{2\mathrm{i}}{ct}\,\mathrm{e}^{-\mathrm{i}t}\sin c-\frac{\mathrm{i}\sqrt{\pi}}{\sqrt{2}t^{3/2}}\,\mathrm{e}^{\frac{1}{4}\mathrm{i}c^2/t}\,F\left(\sqrt{t},\,\frac{c}{2\sqrt{t}}\right). \tag{A.11}$$

Generalizing this answer to the case $b\neq 0$ in the way indicated, and substituting it in the left-hand side of (A.7), we arrive at the desired result.

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